Introduction to Computer Graphics

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Last Time

- Shading models
  - BRDF
  - Lambertian
  - Specular
- Simple lighting calculation
- Tone mapping

Today

- Acceleration data structure
  - How to handle lots of objects

- Light transport simulation
  - Rendering equation

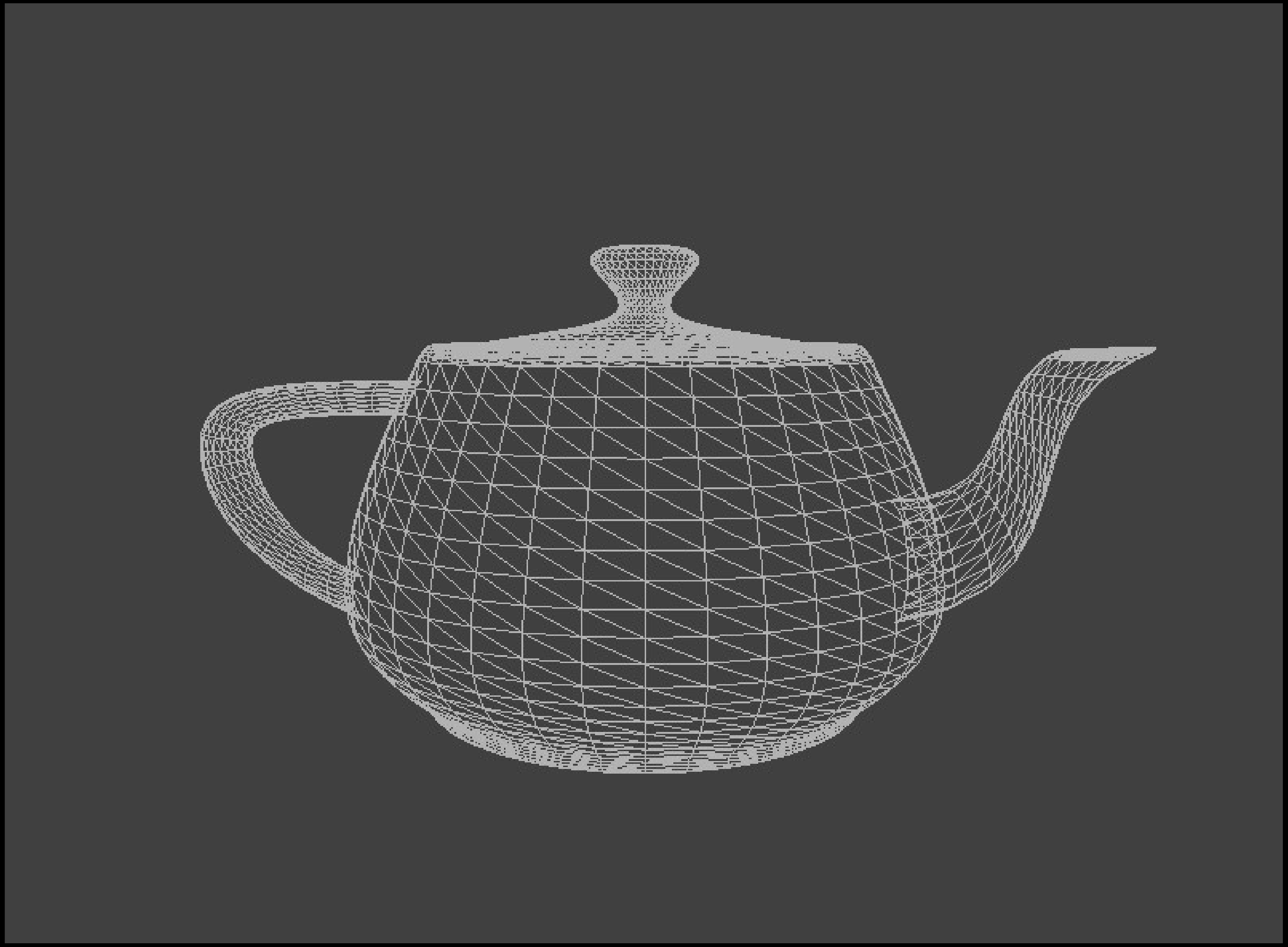
Cost

```
for all pixels {
    ray = generate_camera_ray( pixel )
    for all objects {
        hit = intersect( ray, object )
        if "hit" is closer than "first_hit" {first_hit = hit}
    }
    pixel = shade( first_hit )
}
```

Cost

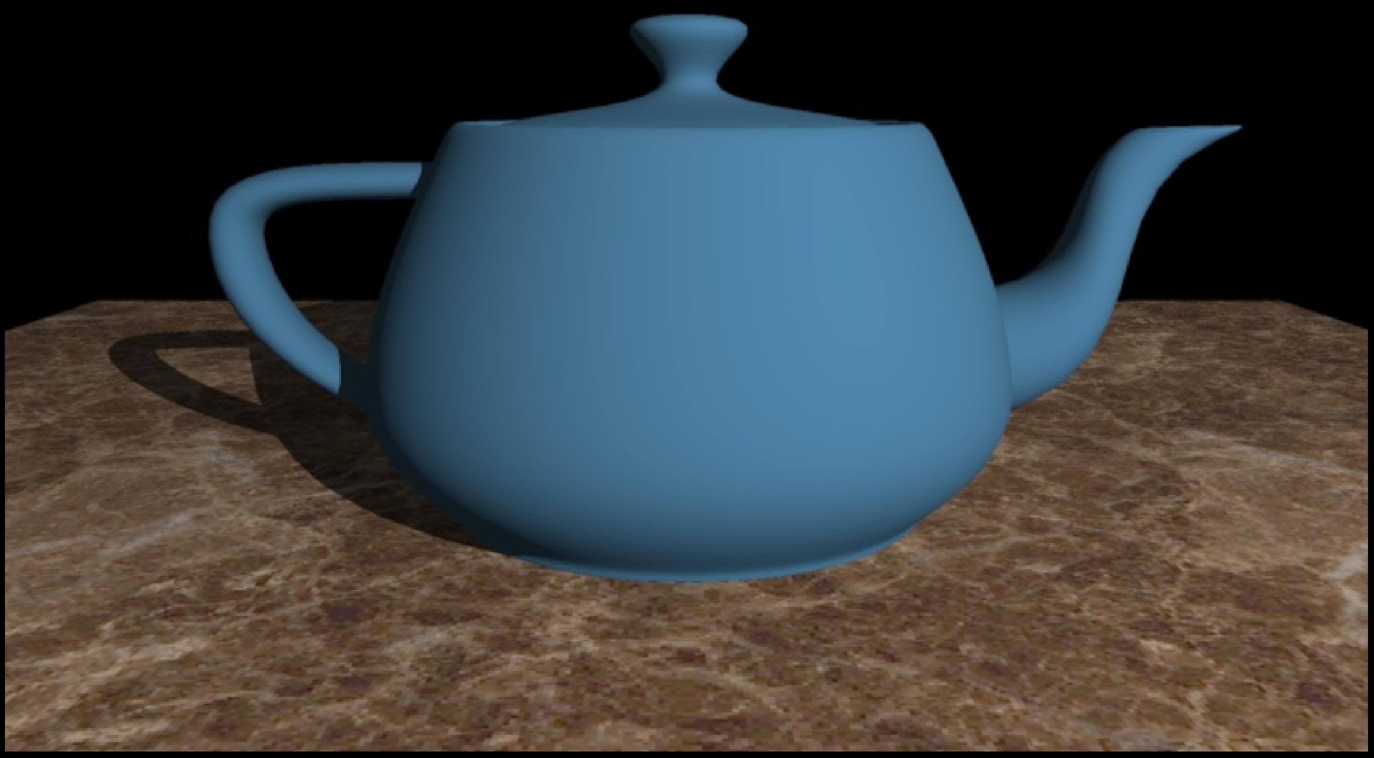
- Number of objects x Number of rays
- Example:
  - 1000 x 1000 image resolution
  - 1000 objects

Many Objects (Triangles)



The teapot has 6320 triangles

Many Objects (Triangles)

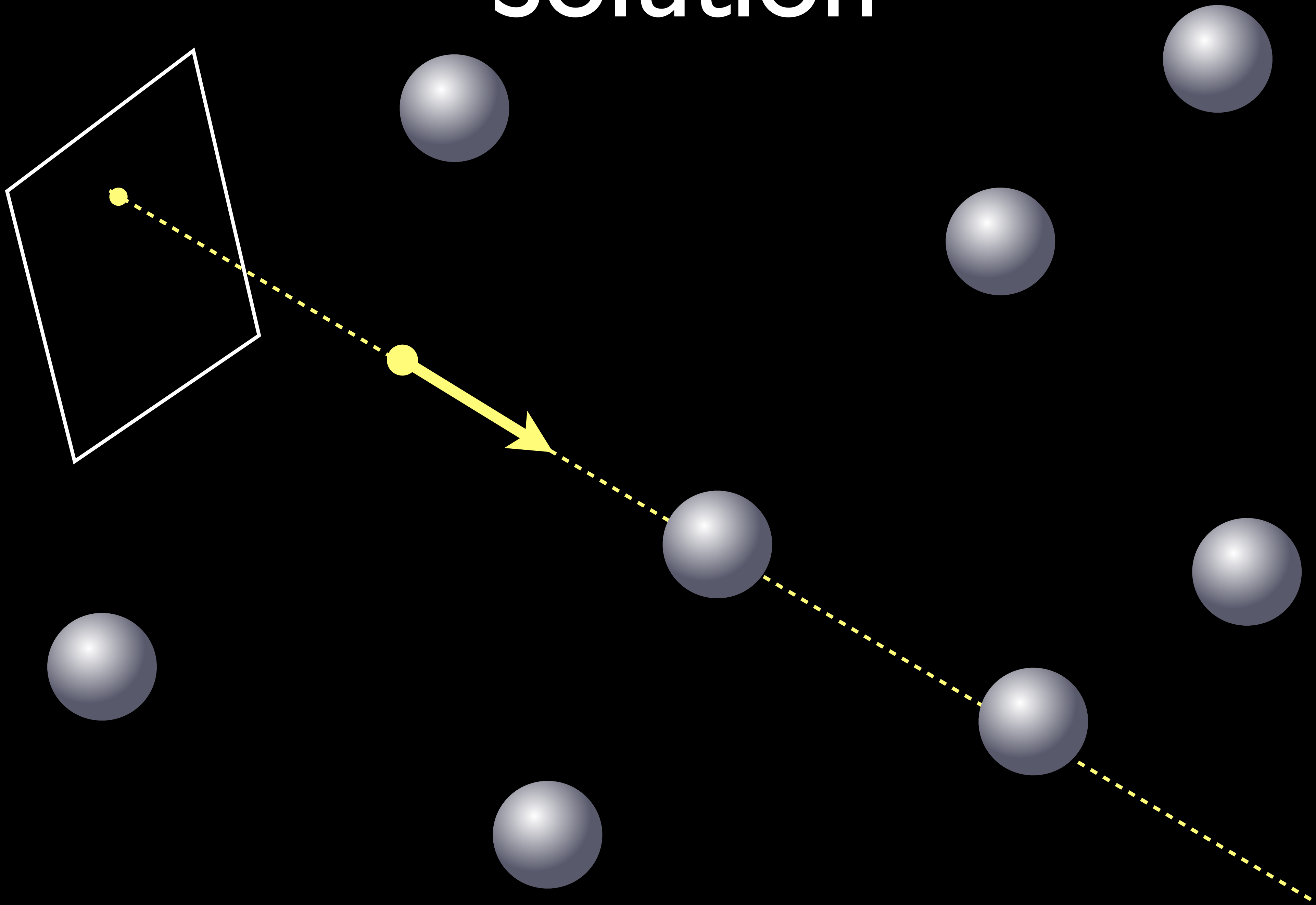


The teapot has 6320 triangles

Solution

- Acceleration data structure
  - Reorganize the list of objects
  - Don’t touch every single object per ray

Solution



Solution

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    }
    pixel = shade( first_hit )
}
```

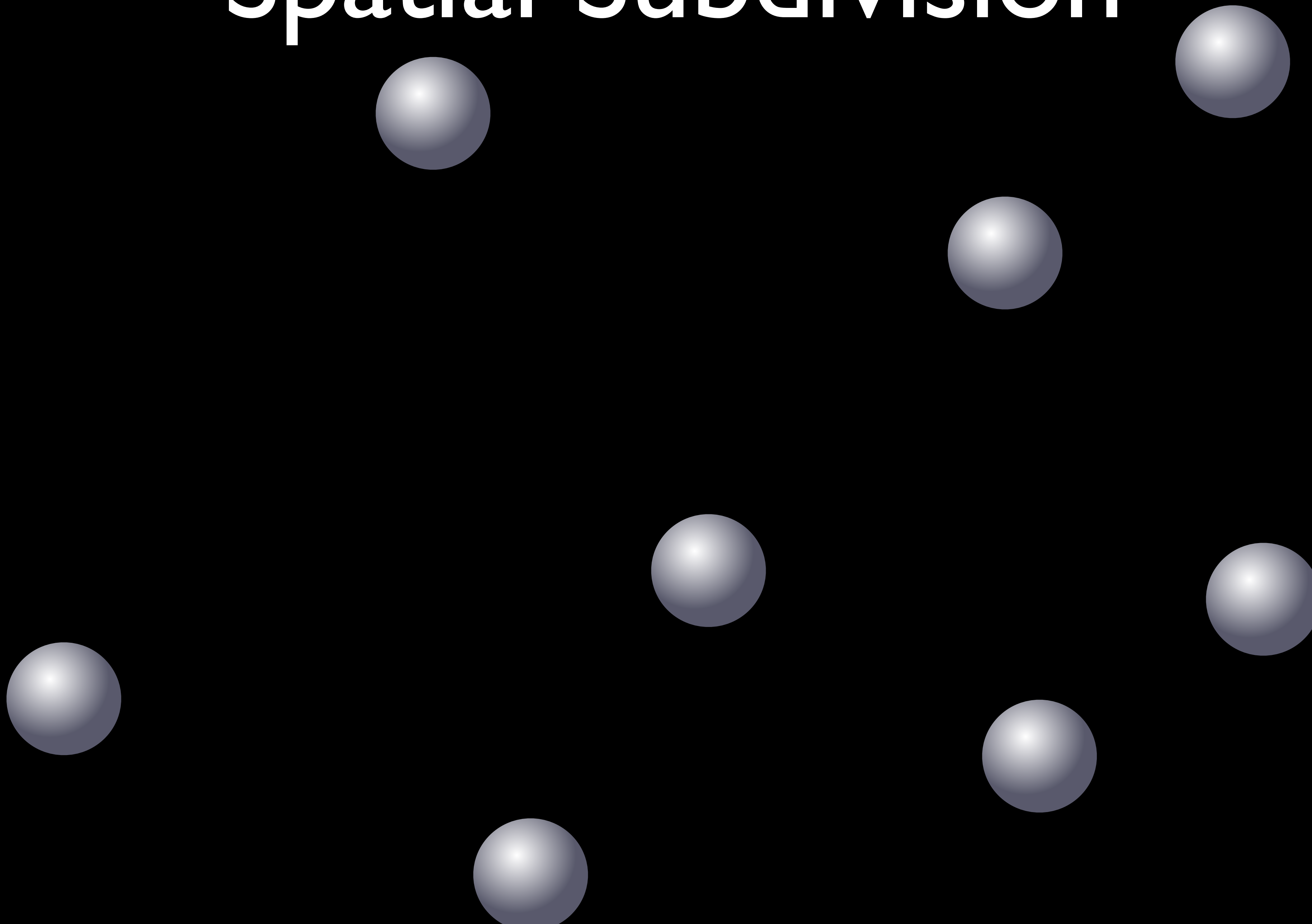
Solution

```
for all pixels {
    ray = generate_camera_ray( pixel )
    first_hit = traverse( ray, accel_data_struct )
    pixel = shade( first_hit )
}
```

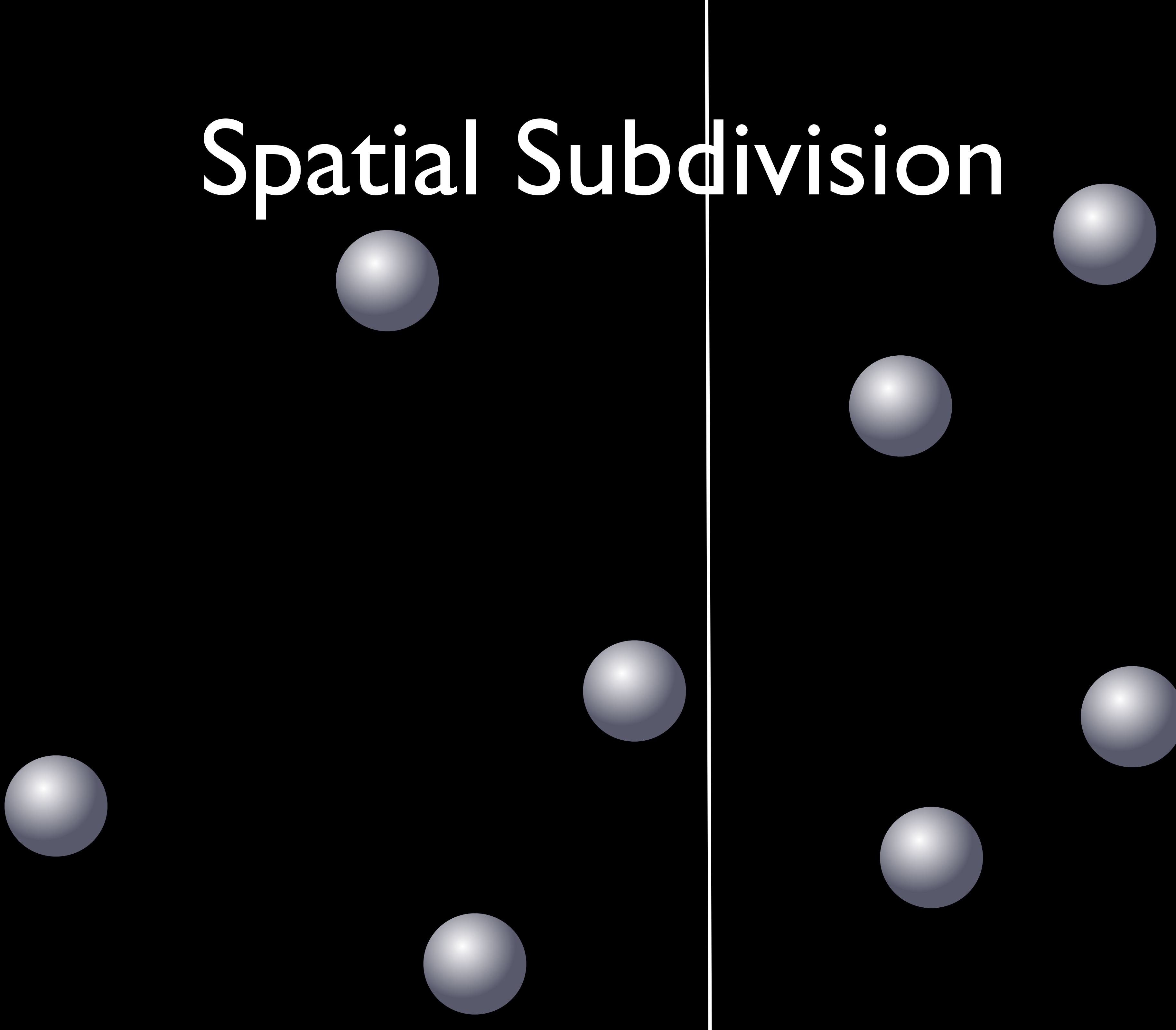
Two Basic Approaches

- Reorganize the space - spatial subdivision
- Reorganize the list - object subdivision

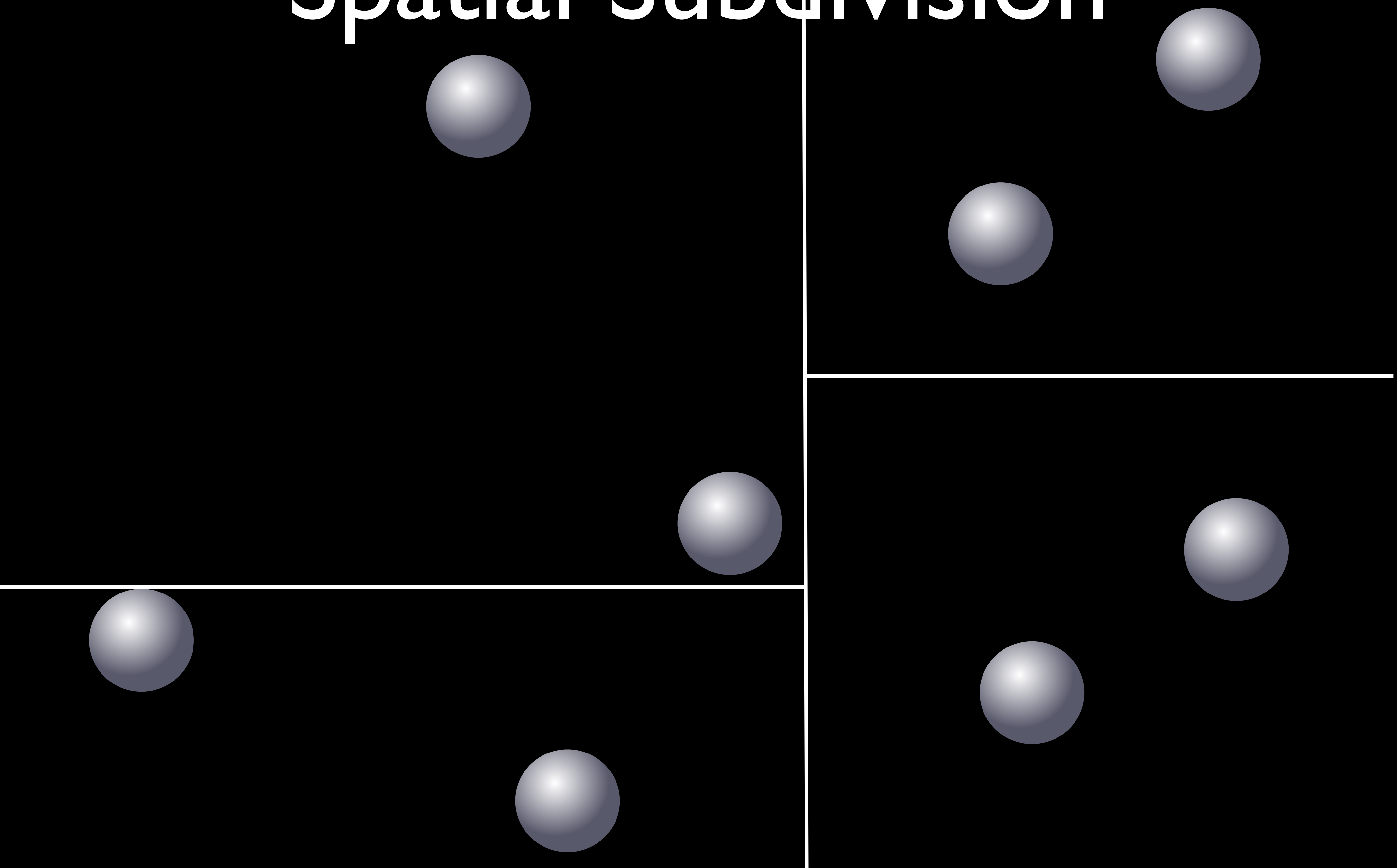
Spatial Subdivision



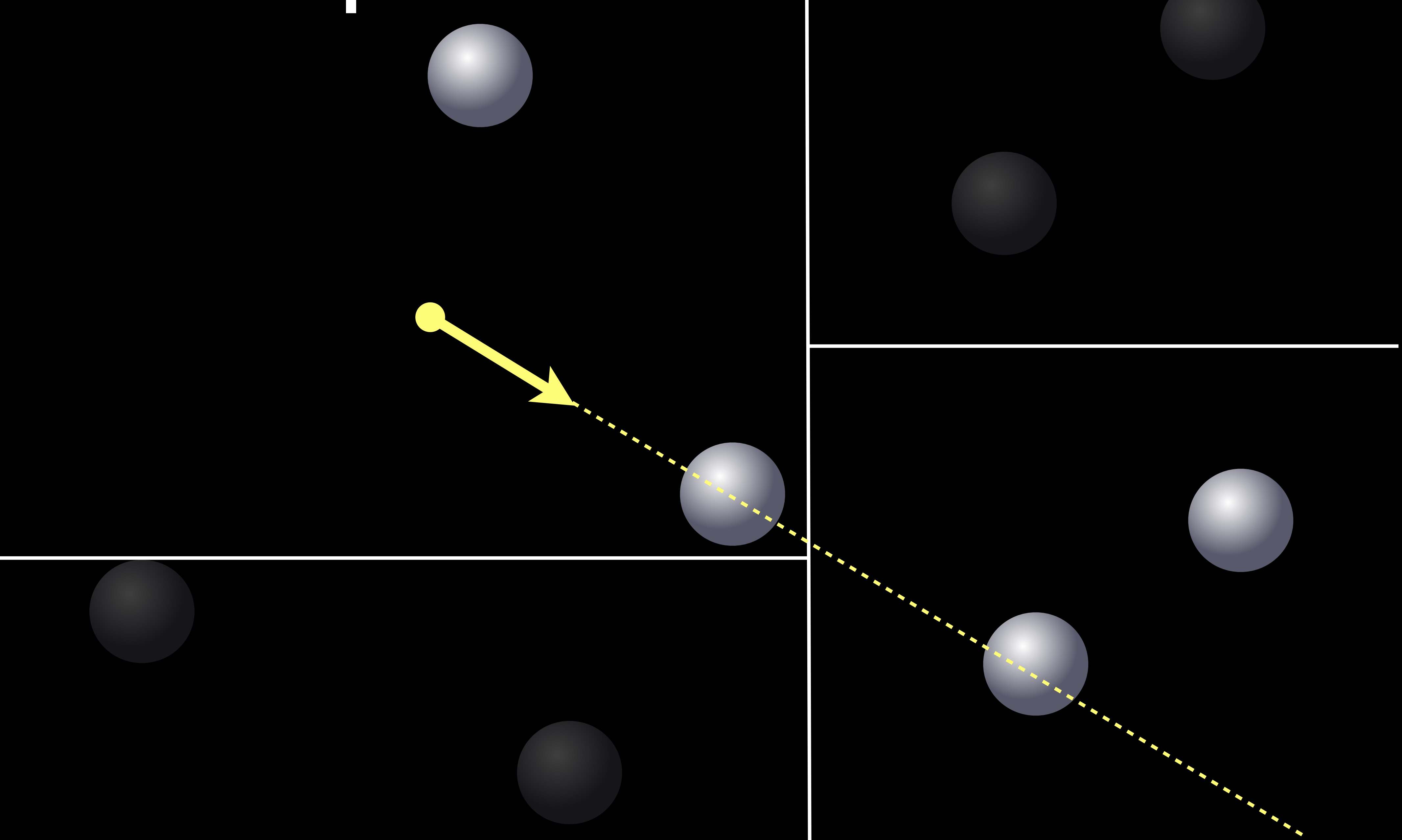
Spatial Subdivision



Spatial Subdivision



Spatial Subdivision



Spatial Subdivision

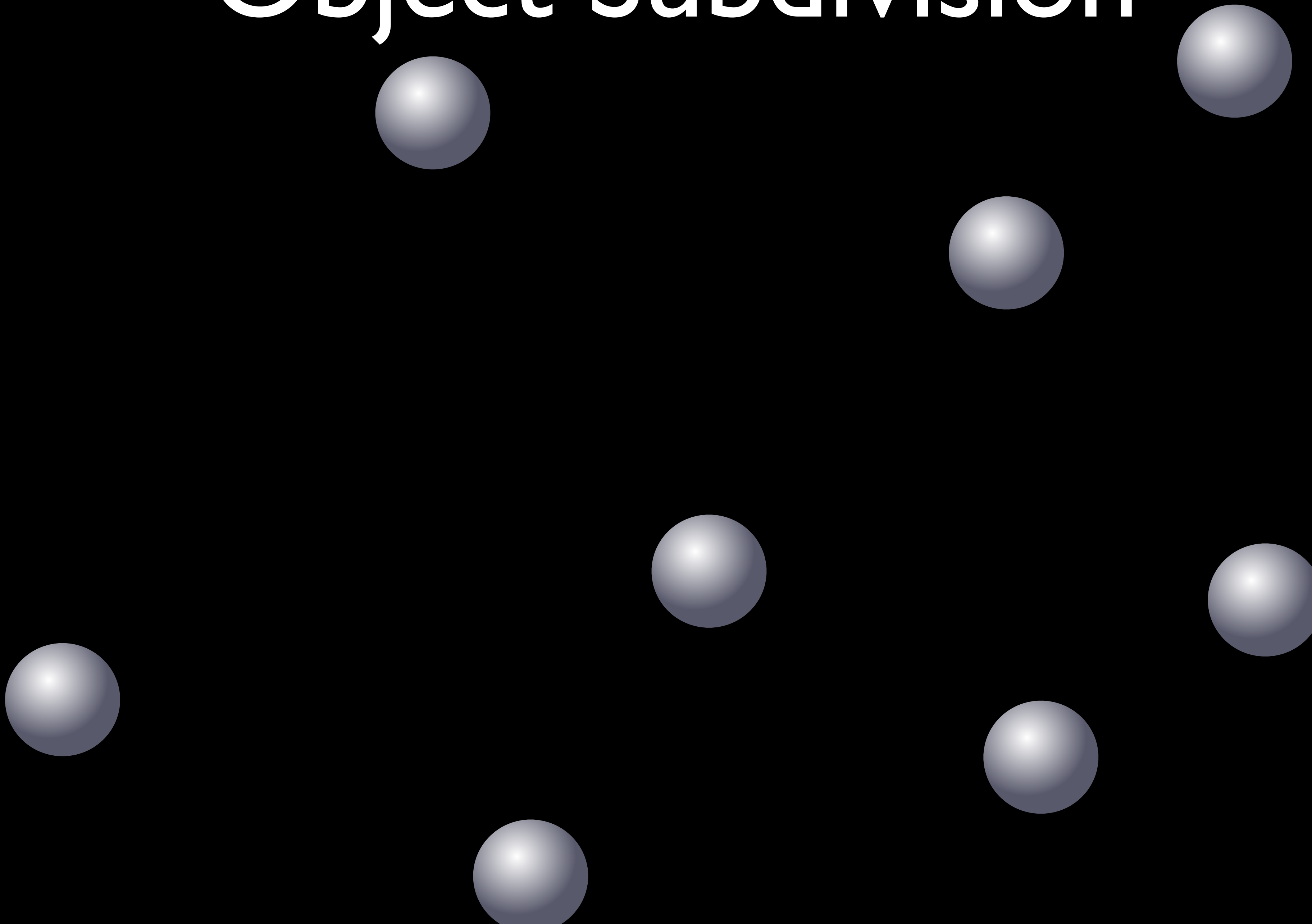
- Hierarchically subdivide the space
  - kD-tree (axis-aligned split)
  - BSP-tree (non-axis-aligned split)

- Each node stores pointers to the subspaces
- Leaf node stores the list of objects that overlap with the subspace

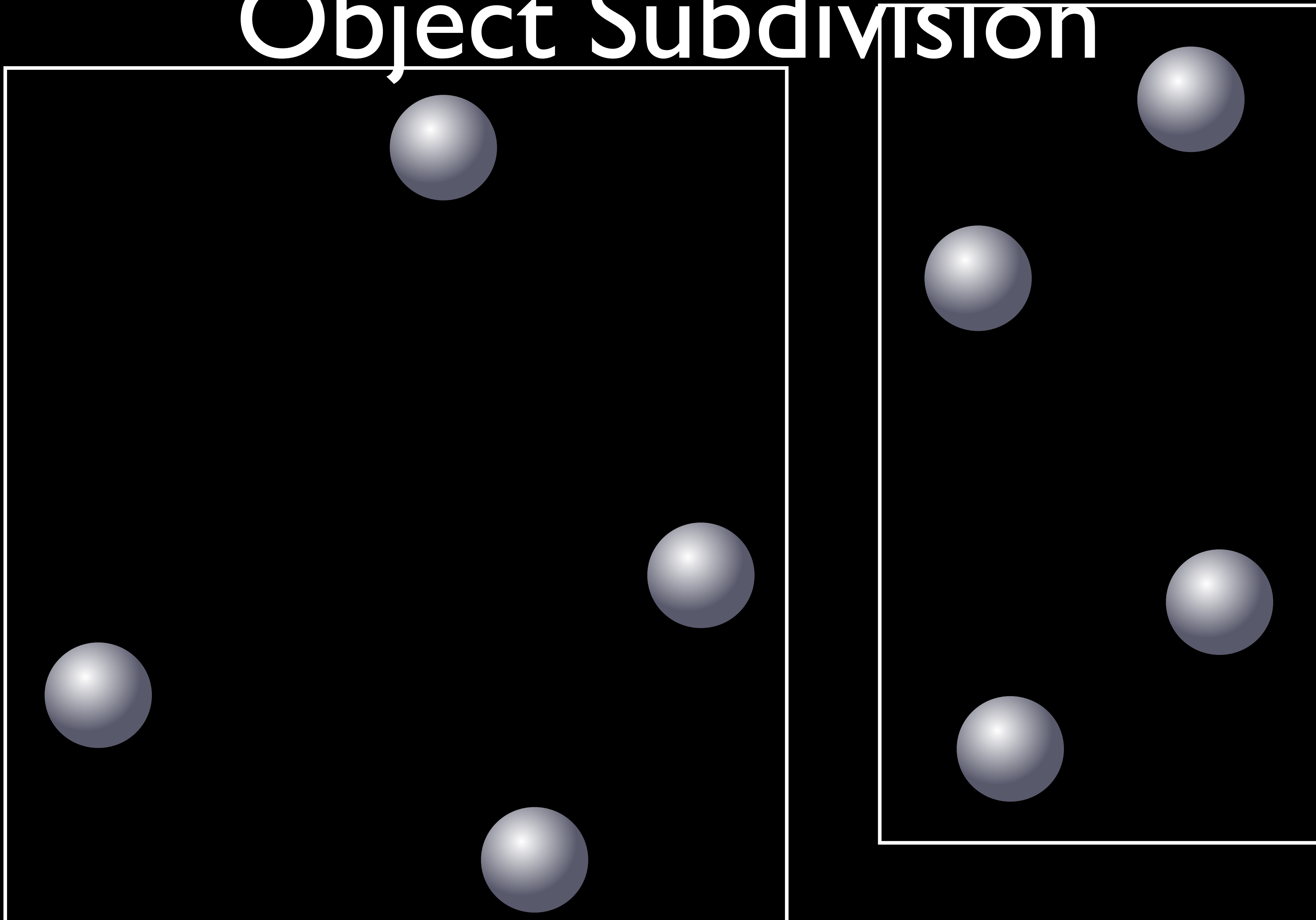
Spatial Subdivision

```
node subdivision( objects, space ) {
    if ( space is small enough ) {
        return make_leaf( objects, space )
    }
    space.split ( &subspace1, &subspace2 )
    for all objects {
        if (overlap(subspace1, object)) objects1.add(object)
        if (overlap(subspace2, object)) objects2.add(object)
    }
    return { subdivision( objects1, subspace1 ),
             subdivision( objects2, subspace2 ) }
}
```

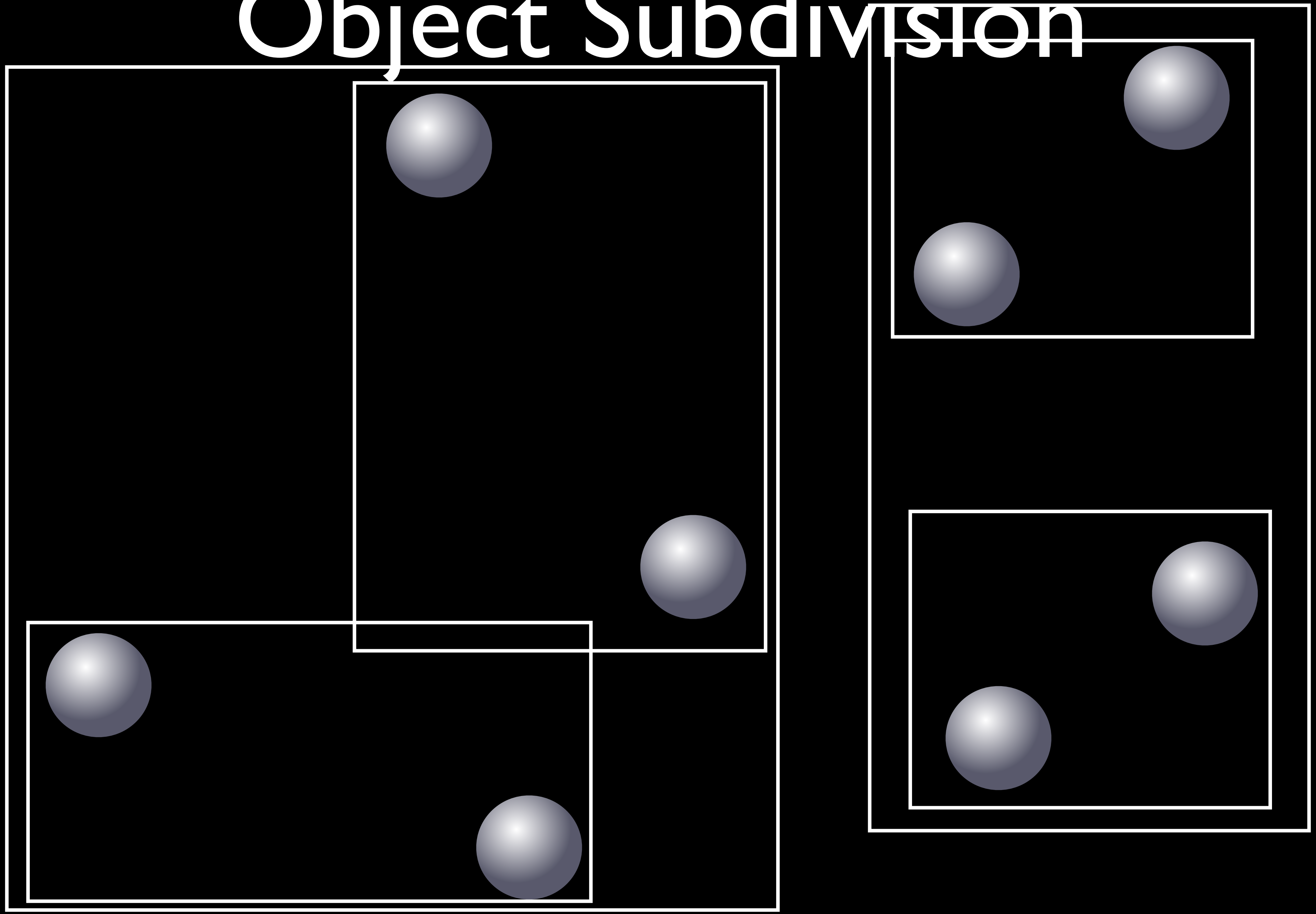
Object Subdivision



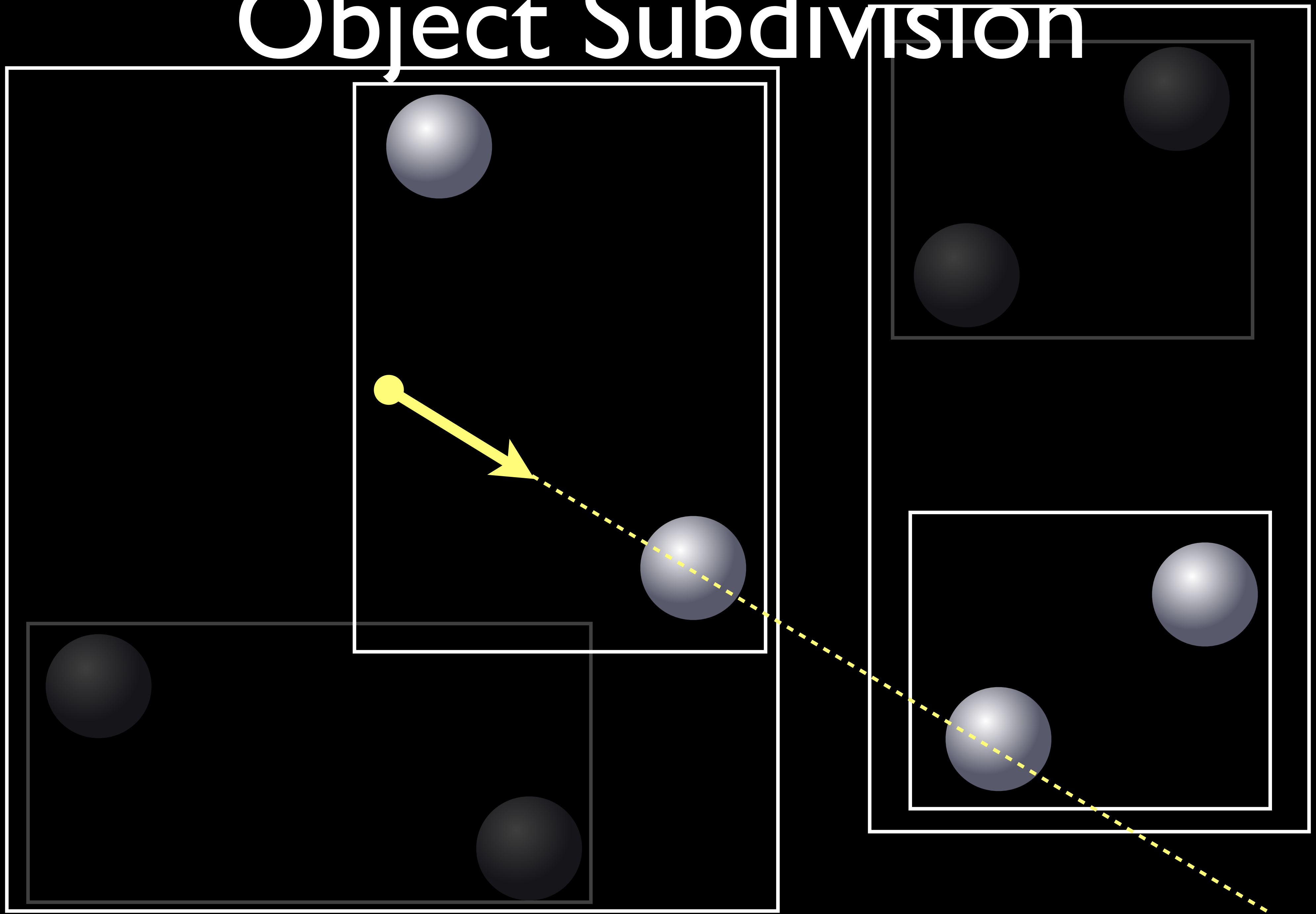
Object Subdivision



Object Subdivision



Object Subdivision



Object Subdivision

- Hierarchically subdivide the list of objects
  - Bounding volume hierarchy (BVH)
  - Choice of volume : sphere, box etc.

- Each node stores pointers to the sublists
- Leaf node stores the list of objects

Object Subdivision

```
node subdivision( objects, space ) {
    if ( number of objects is small enough ) {
        return make_leaf( objects, space )
    }
    objects.split ( &objects1, &object2 )
    subspace1 = bounding_volume( objects1 )
    subspace2 = bounding_volume( objects2 )

    return { subdivision( objects1, subspace1 ),
             subdivision( objects2, subspace2 ) }
}
```

Object vs Spatial

- Two approaches
  - Spatial subdivision (kD-tree)
  - Object subdivision (BVH)
- Still debatable
- Hybrid is possible and explored
- Similar to database queries

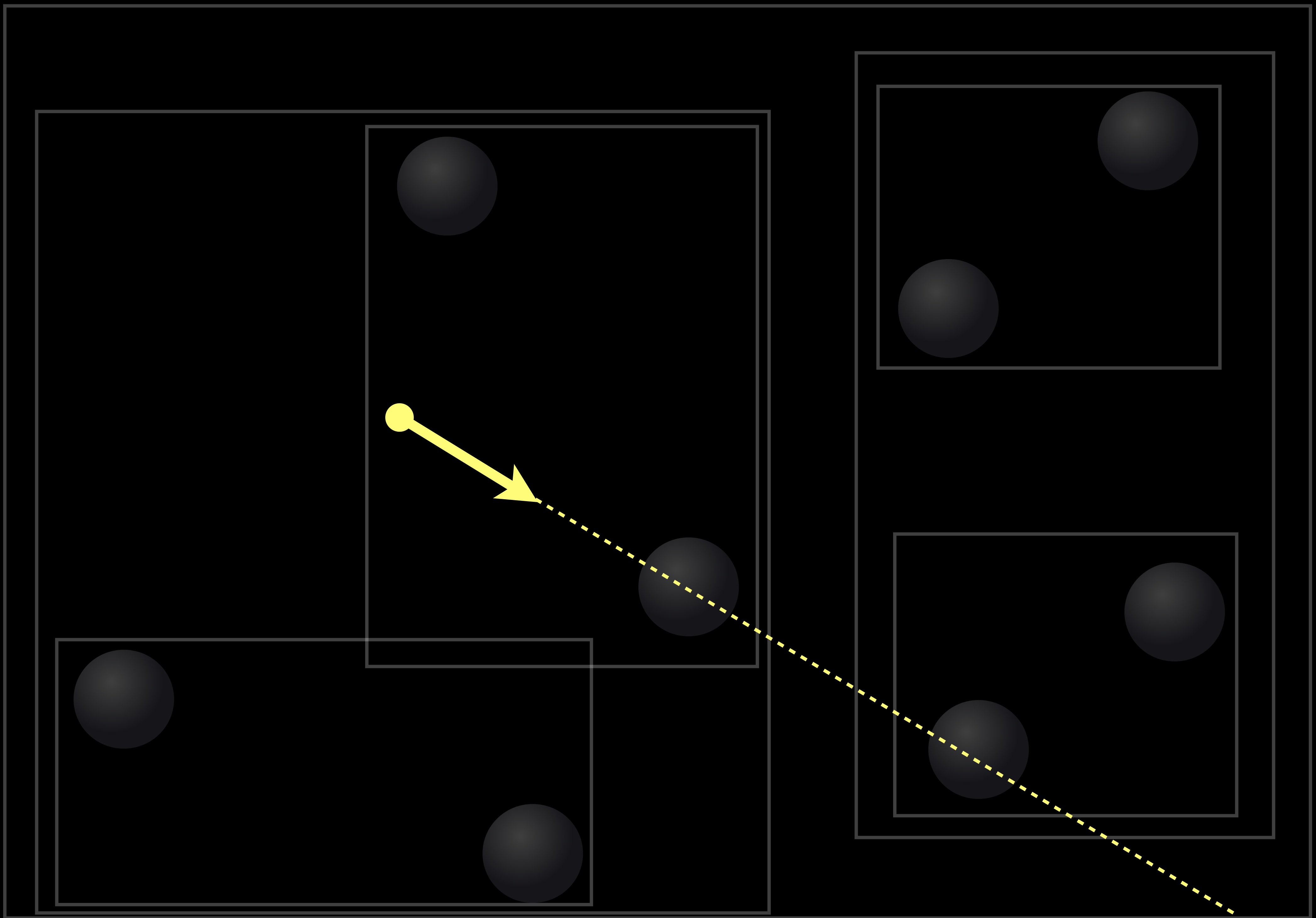
Traversal

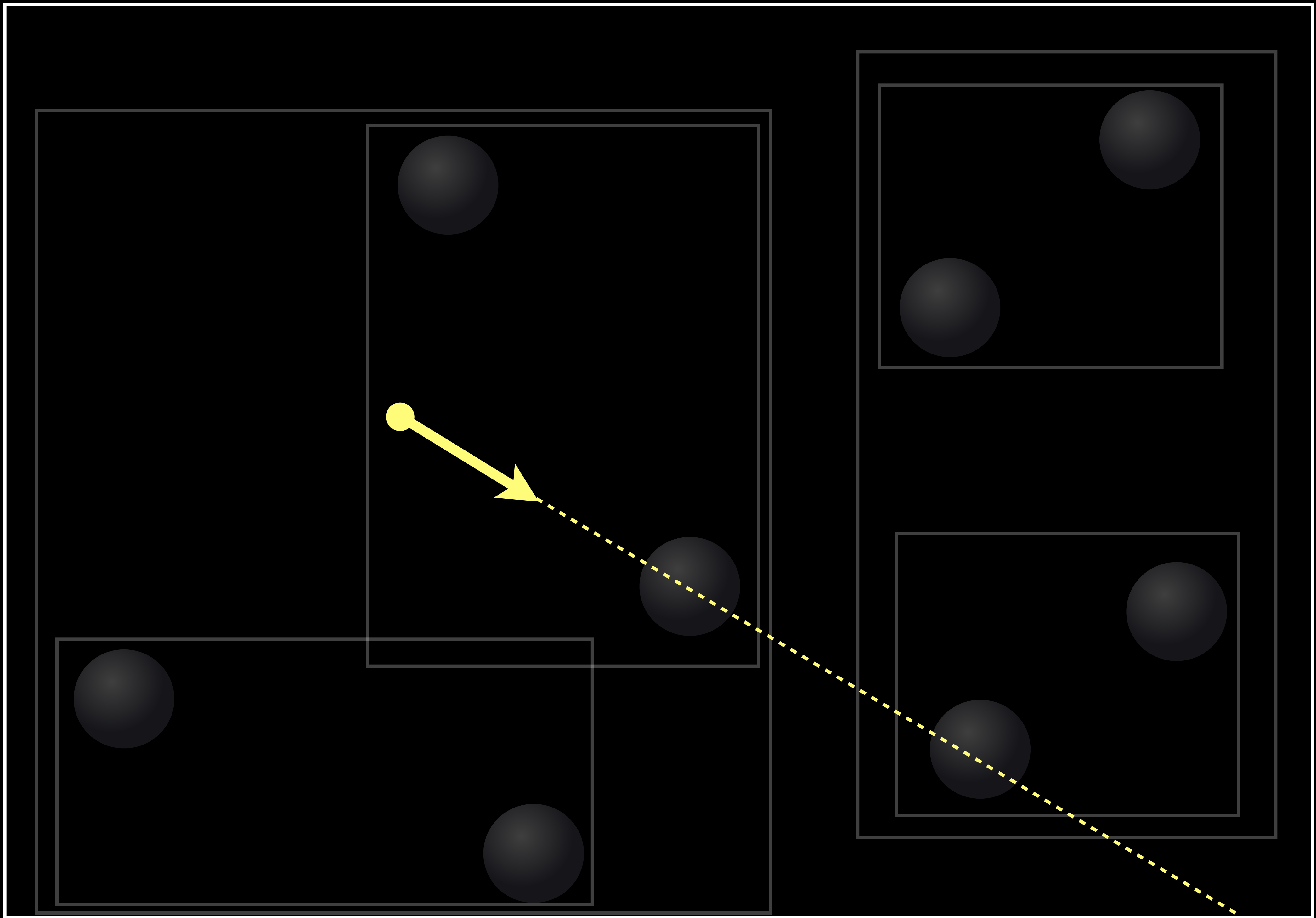
- Goal : Don’t touch every single object
- Given an acceleration data structure
  - Start from the root (entire space)
  - Intersect the ray with child nodes
  - Perform ray-triangle intersections at leaf

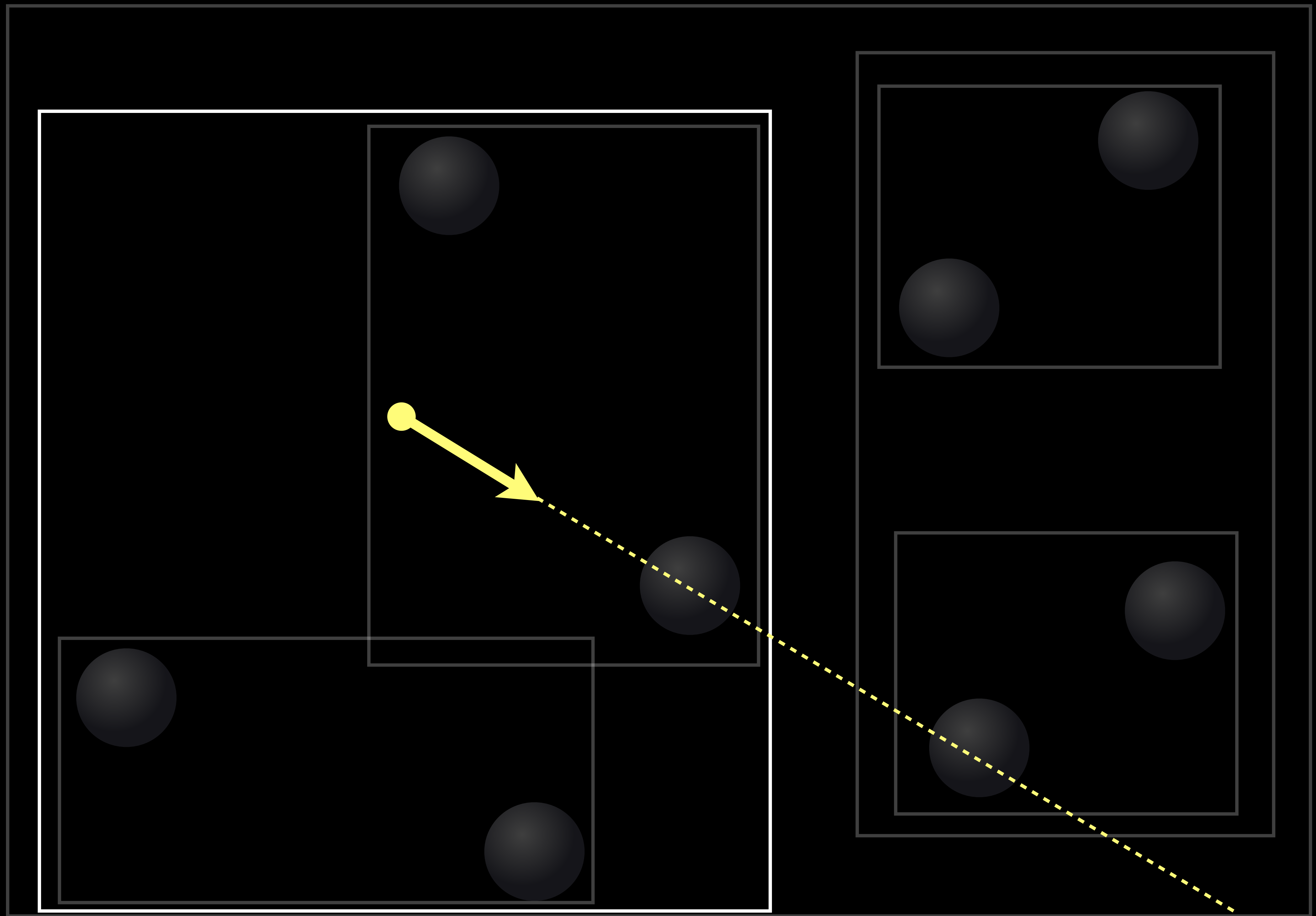
Traversal

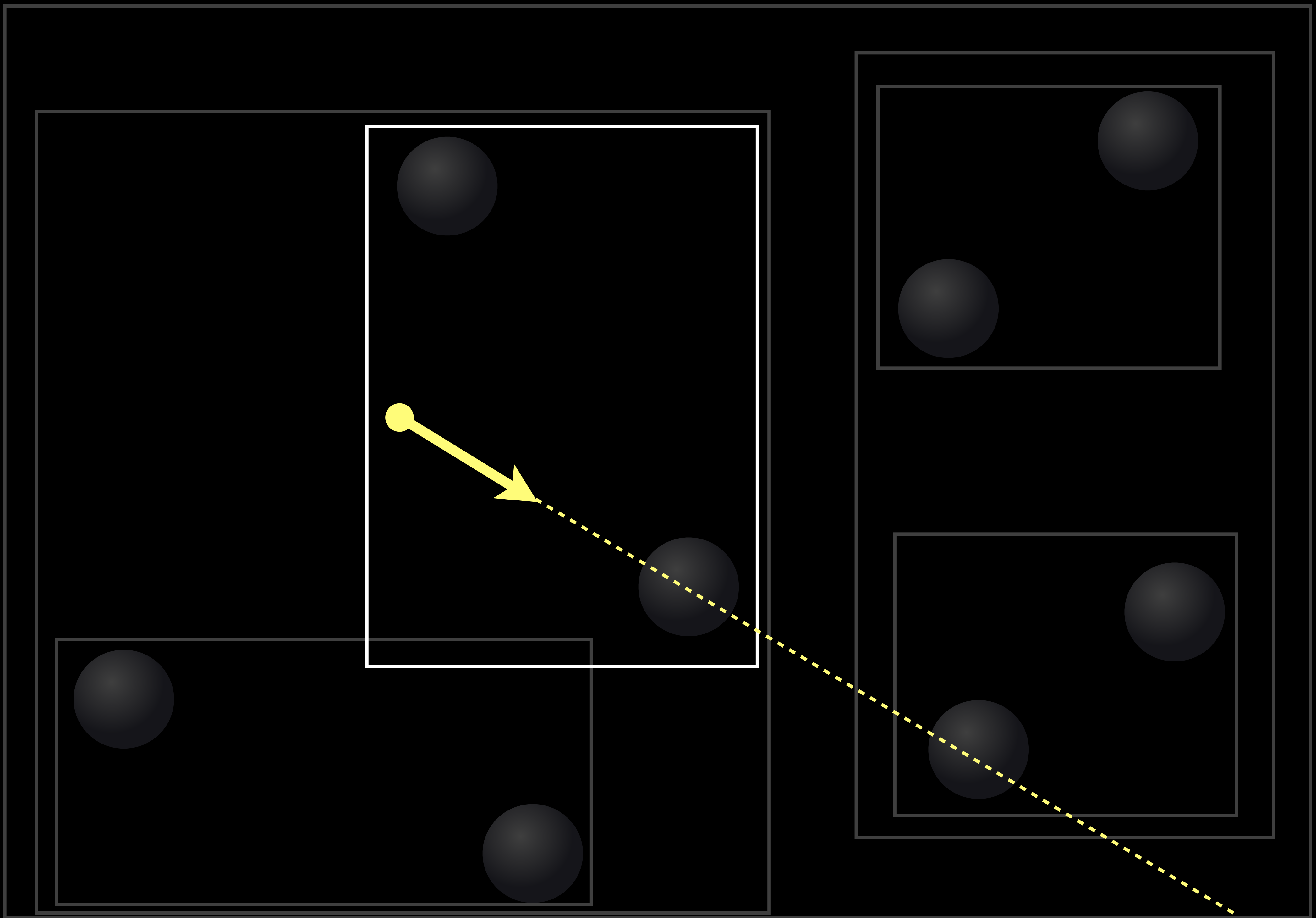
```
hit traverse( ray, node ) {
    if ( IsLeaf( node ) ) {
        for all objects in the node {
            hit = closer( hit, intersect( ray, object ) )
        }
    }
    if (overlap(ray, node.child1)) traverse( ray, node.child1 )
    if (overlap(ray, node.child2)) traverse( ray, node.child2 )
}

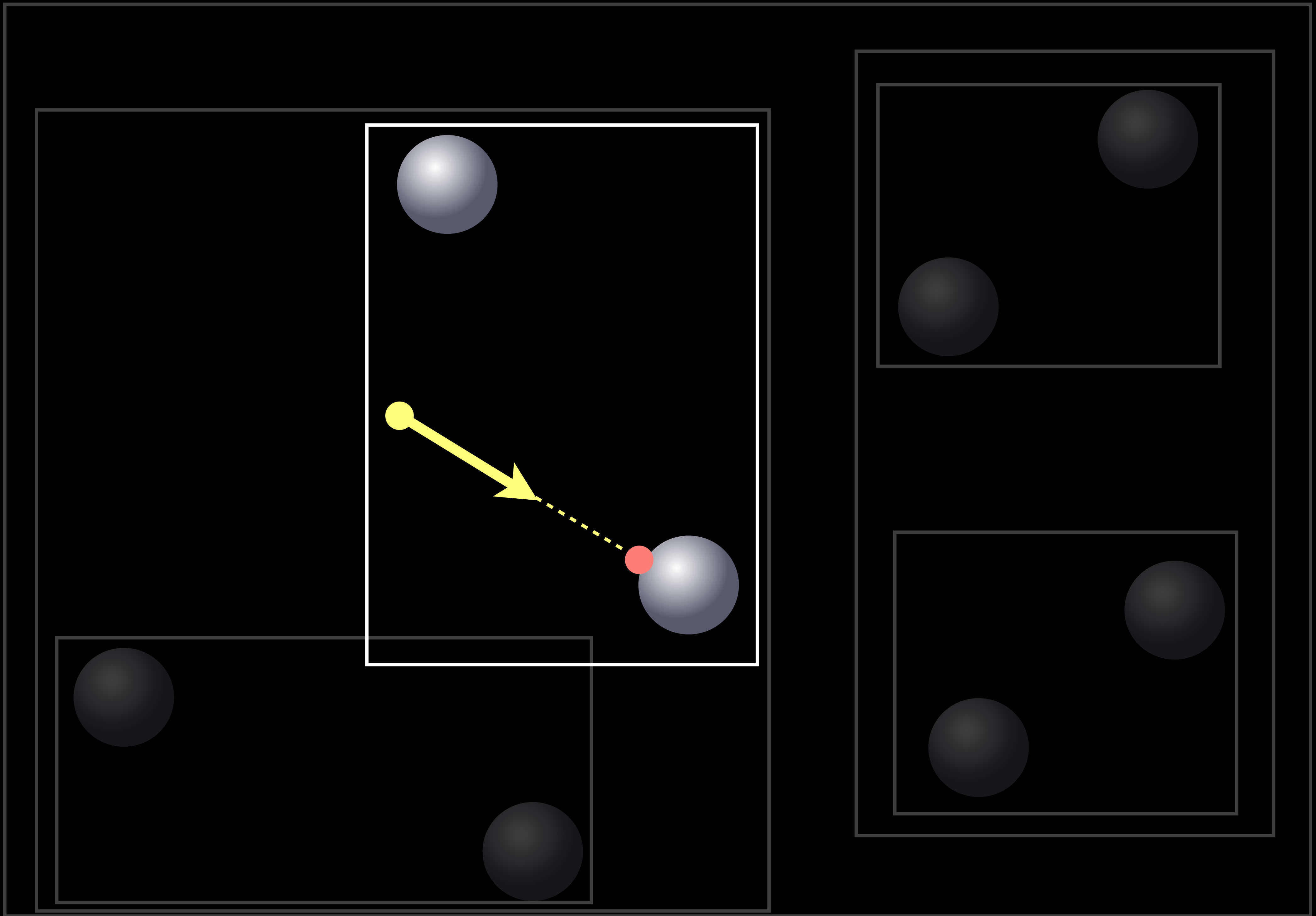
hit = traverse( ray, root )
```

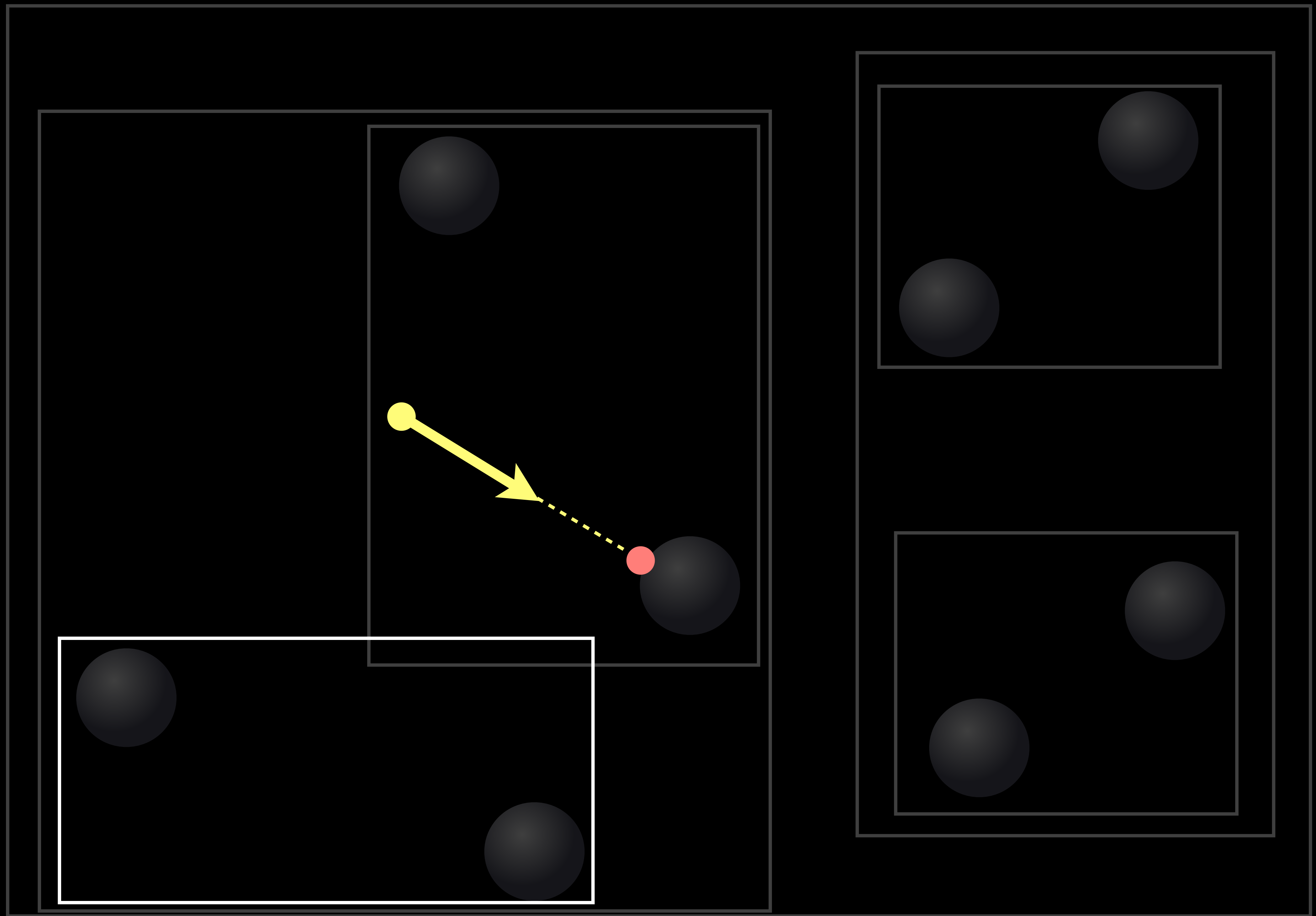


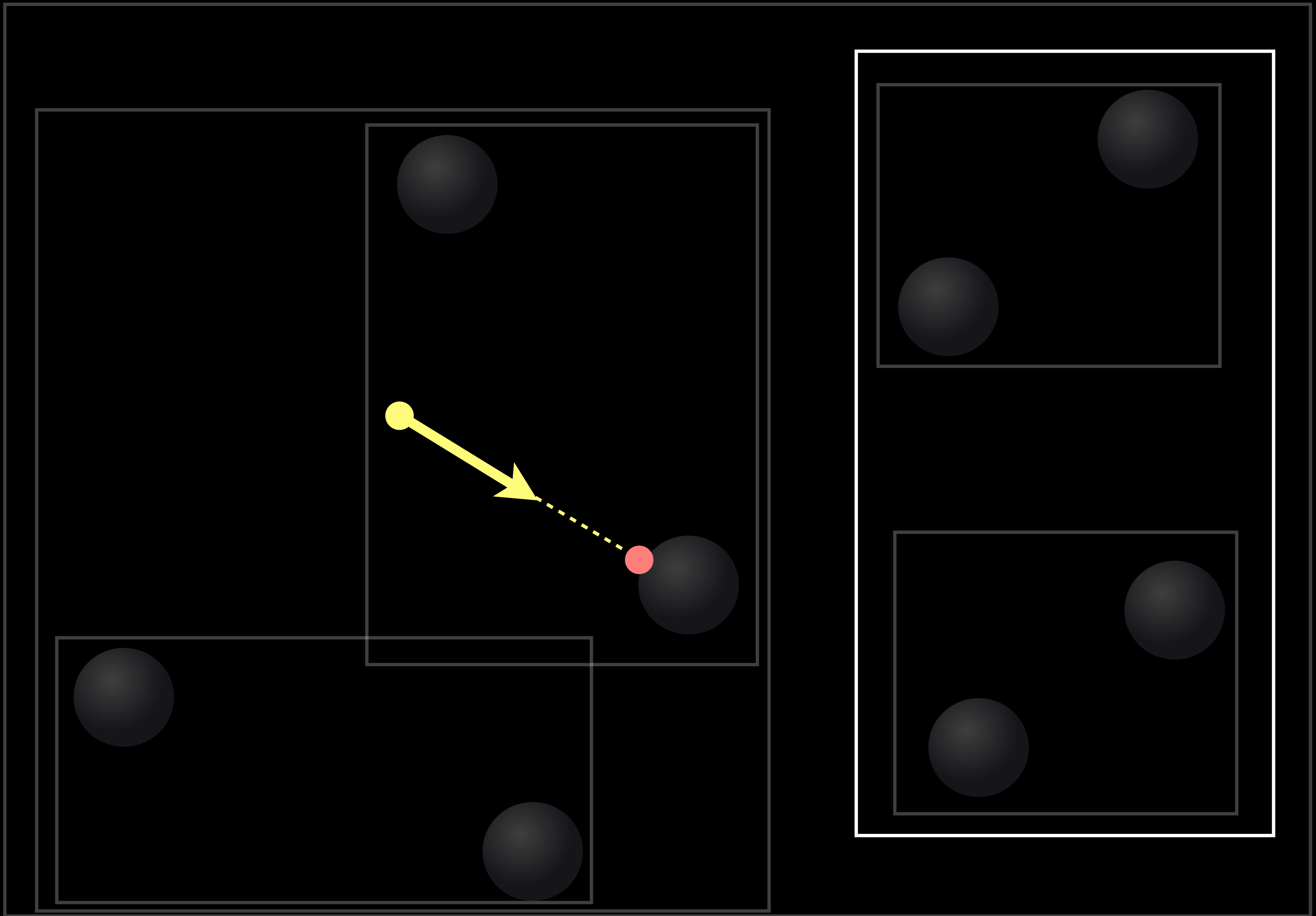


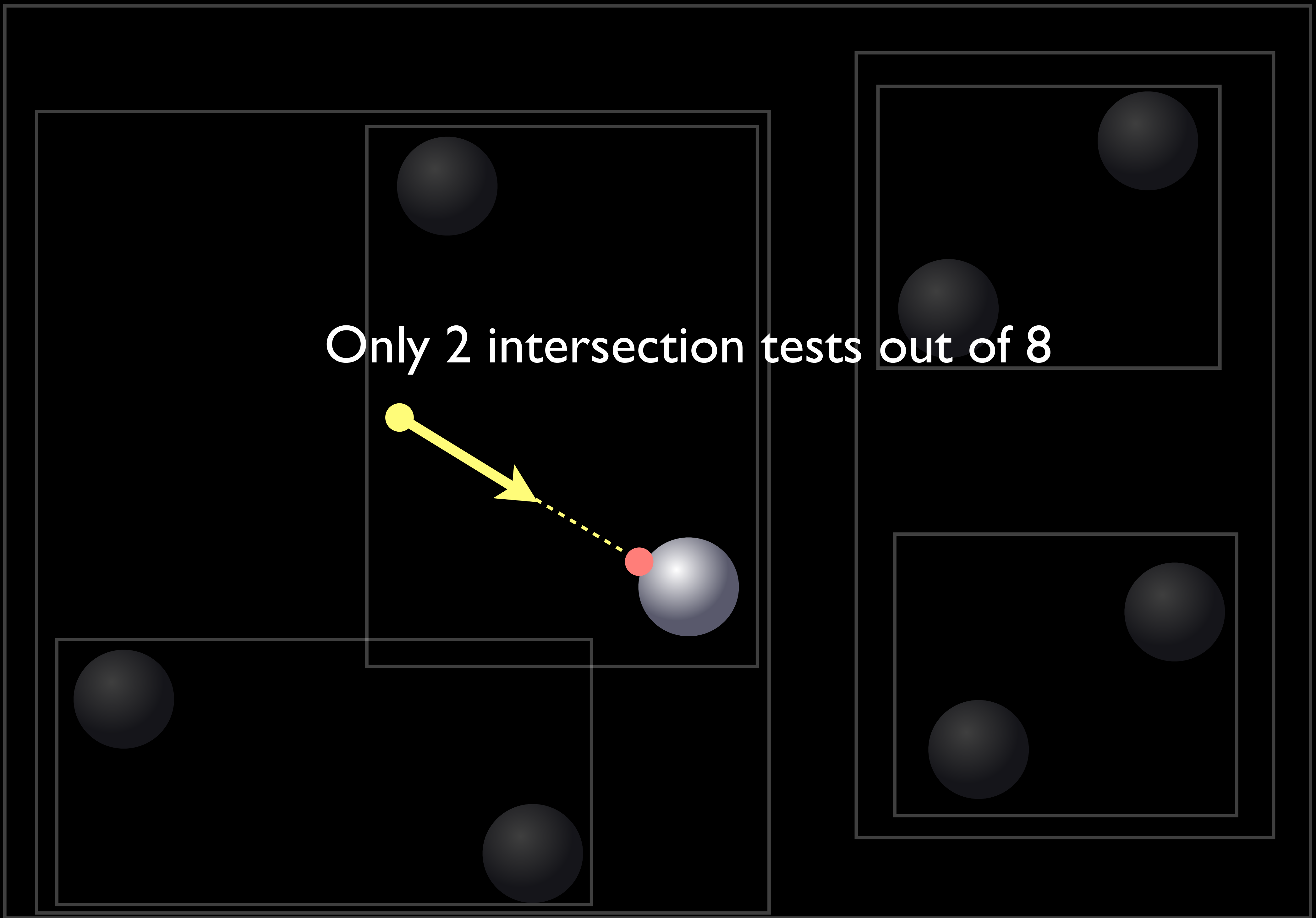











Only 2 intersection tests out of 8

Cost

```
for all pixels {
    ray = generate_camera_ray( pixel )
    first_hit = traverse( ray, accel_data_struct )
    pixel = shade( first_hit )
}
```

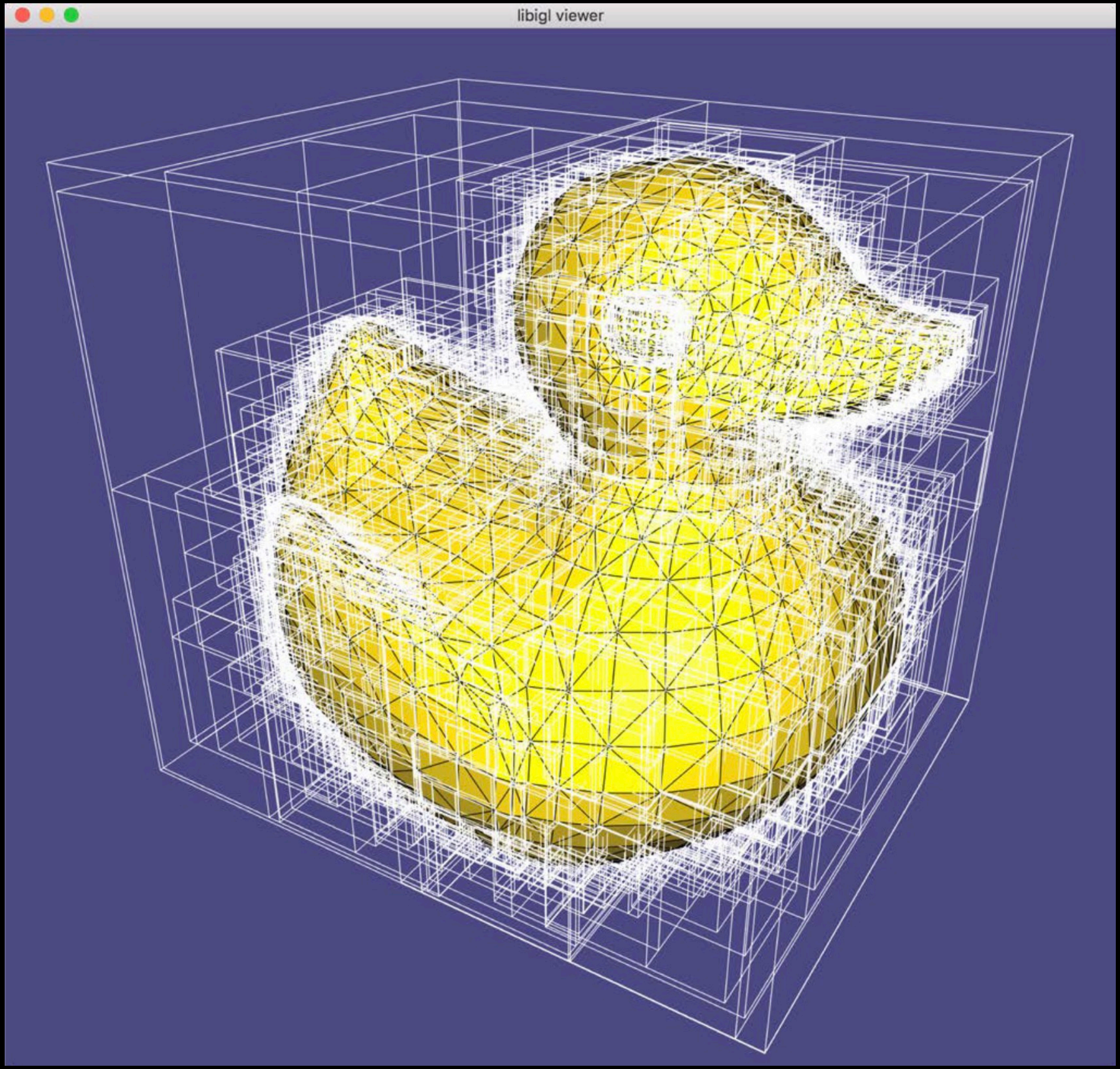
Cost

- No longer as simple as
  "Number of objects x Number of rays"
  - Order analysis is not helpful

- Cost of traversal vs intersection tests
- When should we stop subdivision?

⇒ Surface Area Heuristic (SAH)

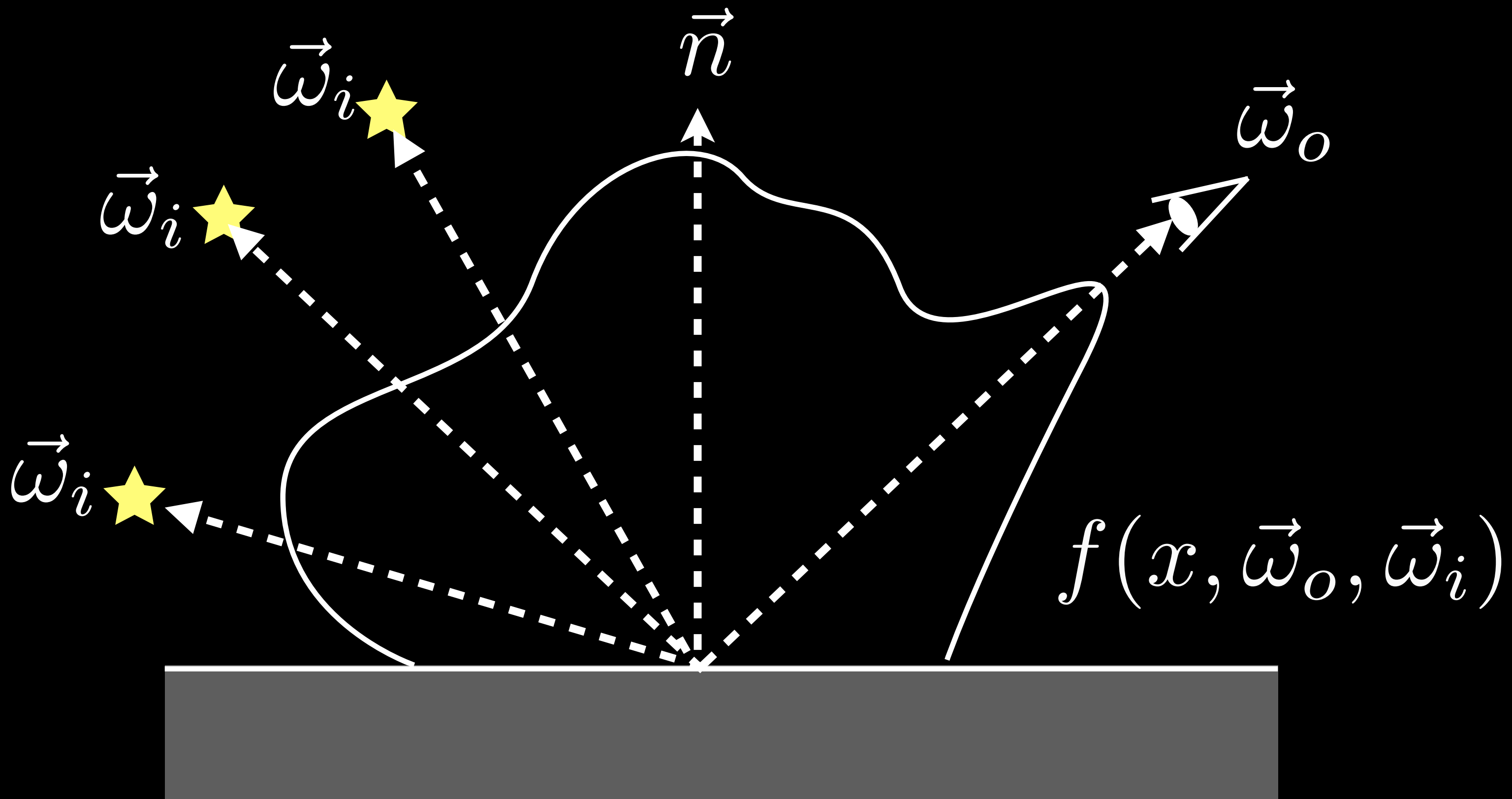
BVH Example



https://github.com/alecjacobson/computer-graphics-bounding-volume-hierarchy

Reflected Radiance

$$L_o(x, \vec{\omega}_o) = \int_\Omega f(x, \vec{\omega}_o, \vec{\omega}_i) L_i(x, \vec{\omega}_i) \cos\theta_i d\omega_i$$

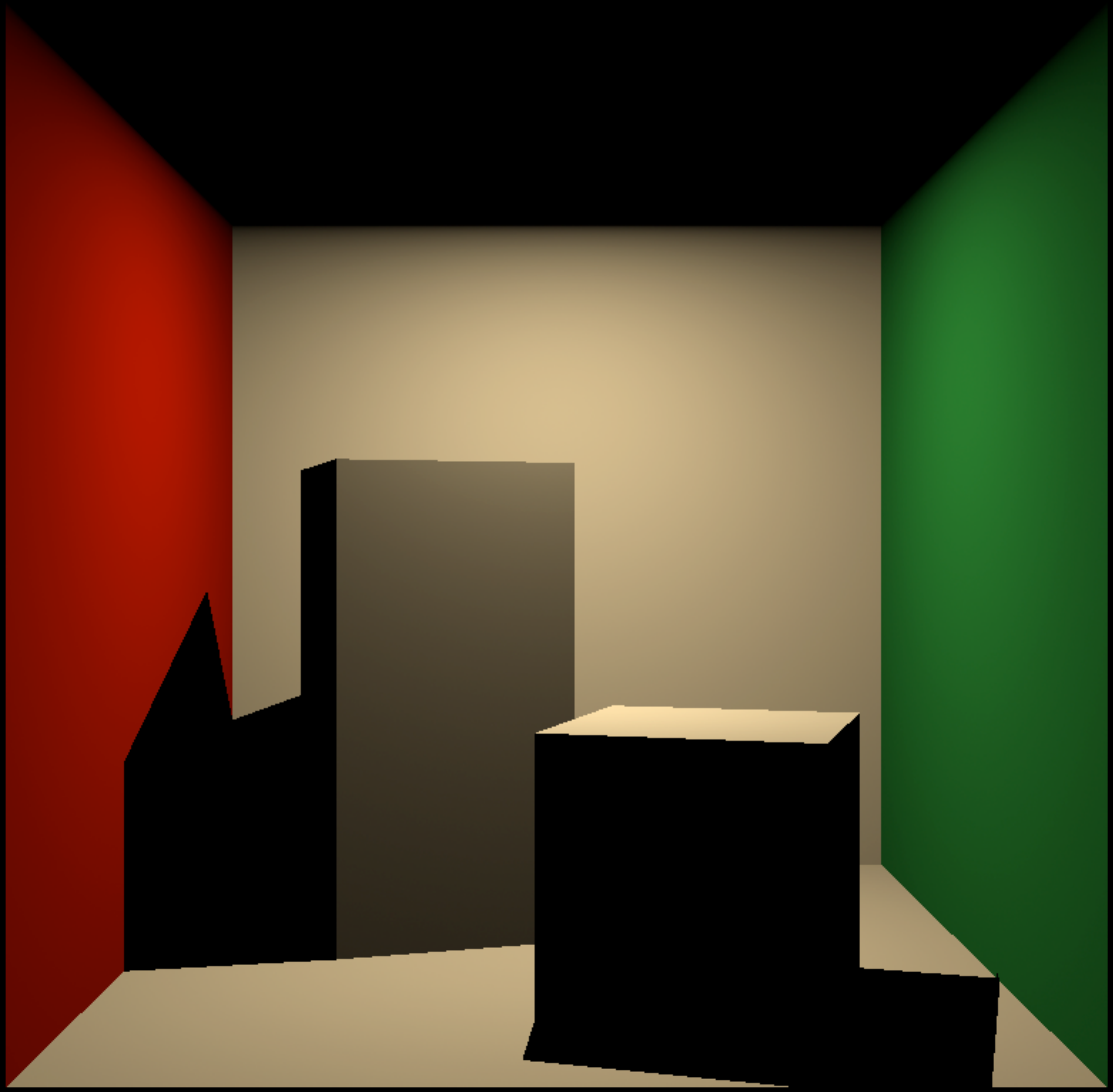


Incident Illumination

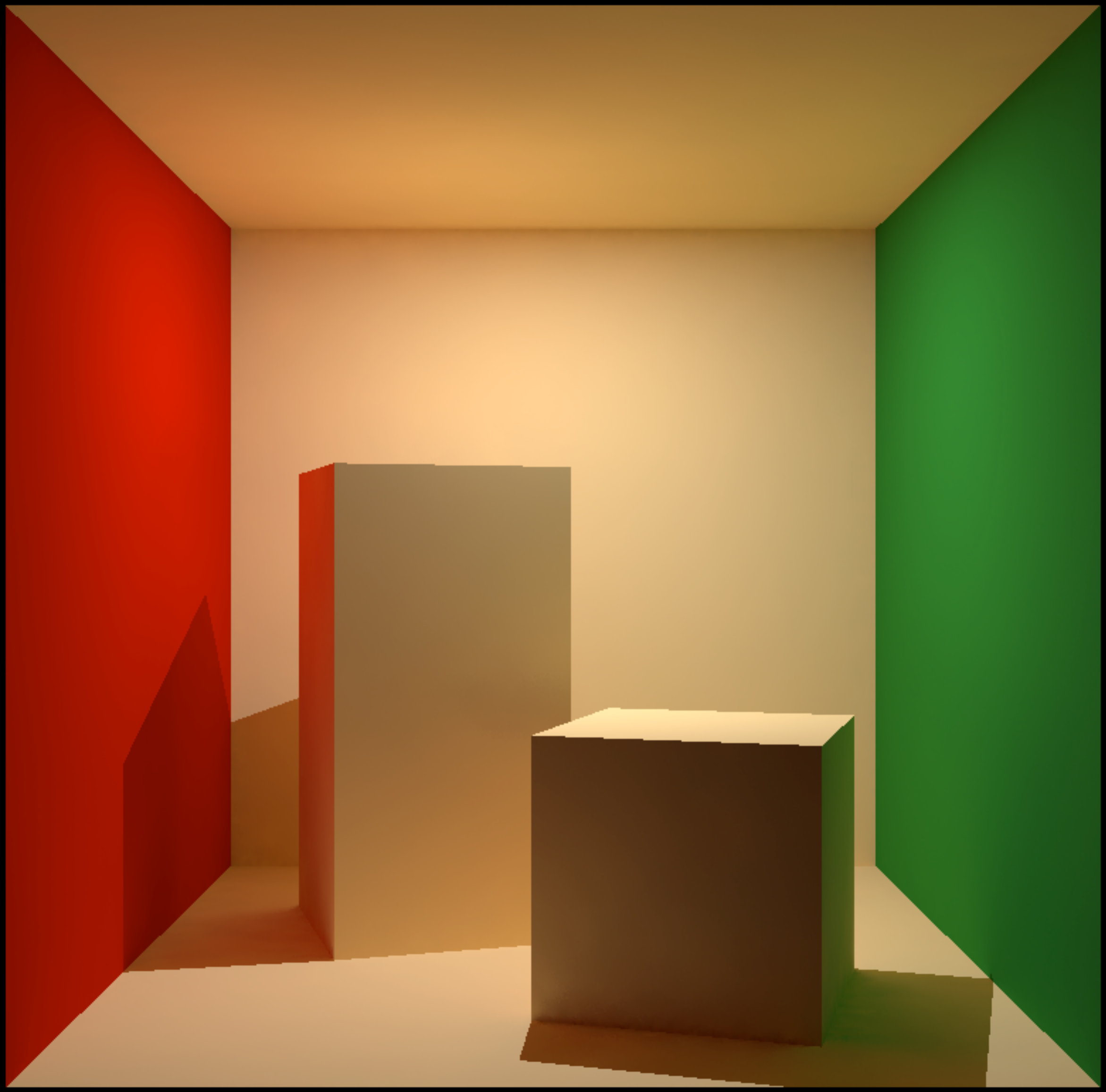
$$L_o(x, \vec{\omega}_o) = \int_{\Omega} f(x, \vec{\omega}_o, \vec{\omega}_i) \boxed{L_i(x, \vec{\omega}_i)} \cos\theta_i d\omega_i$$

- Given by light sources or images

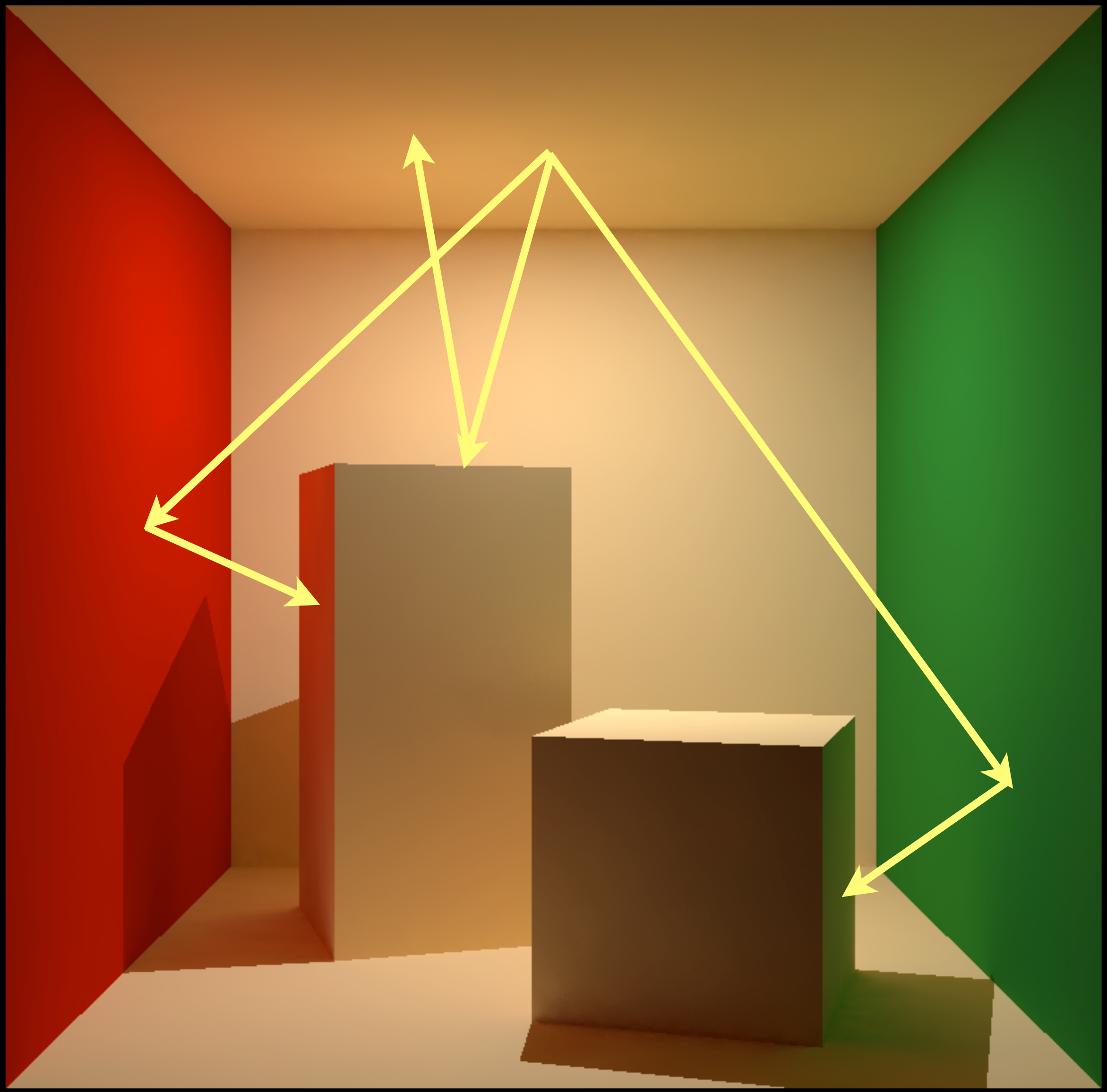
Missing Piece



Missing Piece



Missing Piece



Missing Piece

- Light can bounce off from other surfaces
- Multiple bounces

- Global illumination
  - Other objects affect illumination
  - Shadowing is one example
  - In contrast to local illumination

Transport Operator

- We use the following operator
  - Input: illumination
  - Output: reflected radiance
- Simplified notation

$$L_o(x \to e) = T[L_i]$$

$$\updownarrow$$

$$L_o(x \to e) = \int_\Omega f(l \to x \to e) L_i(l \to x) \cos\theta d\omega$$

Using Transport Operator

- One bounce (i.e., direct)

$$I^0 L_o(x \to e) = T[L_i]$$

- Two bounces

$$I^1 L_o(x \to e) = T[I^0 L_o]$$

Using Transport Operator

- One bounce (i.e., direct)

$$\boxed{I^0 L_o(x \to e)} = T[L_i]$$

- Two bounces

$$I^1 L_o(x \to e) = T[\boxed{I^0 L_o}]$$

Using Transport Operator

- One bounce (i.e., direct)

$$I^0 L_o(x \to e) = T[L_i]$$

- Two bounces

$$\begin{aligned} I^1 L_o(x \to e) &= T[I^0 L_o] \\ &= T[T[L_i]] = T^2[L_i] \end{aligned}$$

Using Transport Operator

- One bounce (i.e., direct)

$$I^0 L_o(x \to e) = T[L_i]$$

- Two bounces

$$\begin{aligned} I^1 L_o(x \to e) &= T[I^0 L_o] \\ &= T[T[L_i]] = T^2[L_i] \end{aligned}$$

$$\boxed{L_o(x \to e) = T[L_i] + T^2[L_i]}$$

Including All Bounces

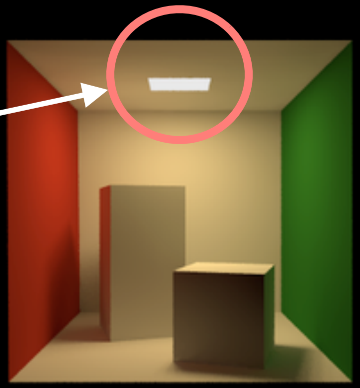
$$L_o(x \to e) = T[L_i] + T^2[L_i] + T^3[L_i] + \cdots$$

Including All Bounces

$$L_o(x \rightarrow e) = \boxed{T[L_i]} + \boxed{T^2[L_i]} + \boxed{T^3[L_i]} + \cdots$$

Direct illumination   Two bounces   Three bounces

Including All Bounces



Directly visible light sources

$$L_o(x \to e) = \boxed{L_i} + \boxed{T[L_i]} + \boxed{T^2[L_i]} + \boxed{T^3[L_i]} + \cdots$$

Direct illumination Two bounces Three bounces

To the Rendering Equation

- Remember the Neumann series

$$\frac{I}{I-K} = I + K + K^2 + K^3 + \cdots$$

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$$\begin{aligned} L_o(x \to e) &= L_i + T[L_i] + T^2[L_i] + T^3[L_i] + \cdots \\ &= \frac{I}{I-T}[L_i] \end{aligned}$$

To the Rendering Equation

- Remember the Neumann series

$$\frac{I}{I-K} = I + K + K^2 + K^3 + \cdots$$

$$\begin{aligned} L_o(x \to e) &= L_i + T[L_i] + T^2[L_i] + T^3[L_i] + \cdots \\ &= \frac{I}{I-T}[L_i] \end{aligned}$$

$$(I - T)[L_o(x \to e)] = L_i$$

To the Rendering Equation

$$(I - T)[L_o(x \to e)] = L_i$$

$$L_o(x \to e) - T[L_o(x \to e)] = L_i$$

$$L_o(x \to e) = L_i + T[L_o(x \to e)]$$

$$L(x \to e) = L_i(x \to e) + \int_\Omega f(\omega, x \to e) L(\omega) \cos\theta d\omega$$

To the Rendering Equation

$$(I - T)[L_o(x \to e)] = L_i$$

$$L_o(x \to e) - T[L_o(x \to e)] = L_i$$

$$L_o(x \to e) = L_i + T[L_o(x \to e)]$$

$$\boxed{L(x \to e) = L_i(x \to e) + \int_\Omega f(\omega, x \to e) L(\omega) \cos\theta d\omega}$$

Rendering Equation

Rendering Equation

- Describe the equilibrium of radiance
- Rendering algorithms = solvers of R.E.
  - James Kajiya in 1986

$$L(x \to e) = \underbrace{\boxed{L_i(x \to e)}}_{\substack{\text{Self-emission} \\ \text{(zero if x is not light source)}}} + \int_\Omega \underbrace{\boxed{f(\omega, x \to e)}}_{\text{BRDF}} L(\omega) \cos\theta d\omega$$

directions for making pictures using numbers
(explained using only the ten hundred words people use most often)

position on the stuff

direction towards the eye

light made by the stuff (sometimes because it is very hot)

the answer to how much light from an interesting direction that will keep going in the direction towards the eye, after hitting stuff at the position (this is easy for mirrors, not so easy for everything else)

the light that comes from an interesting direction towards the position on the stuff

how much the light becomes less bright because the stuff leans away from the interesting direction

$$L_o(x,\ \omega_o) = L_e(x,\ \omega_o) + \int_\Omega f_r(x,\ \omega_i \rightarrow \omega_o)\ L_i(x,\ \omega_i)\ (\omega_i \cdot n)\ d\omega_i$$

light that leaves the position on the stuff and reaches the eye

light can be added said a man who sat under a tree many years ago

for lots of interesting directions inside half a ball facing up from the stuff, add up all the answers in between

this idea came from http://xkcd.com/1133/

@levork

https://twitter.com/levork/status/609603797258600448/photo/1

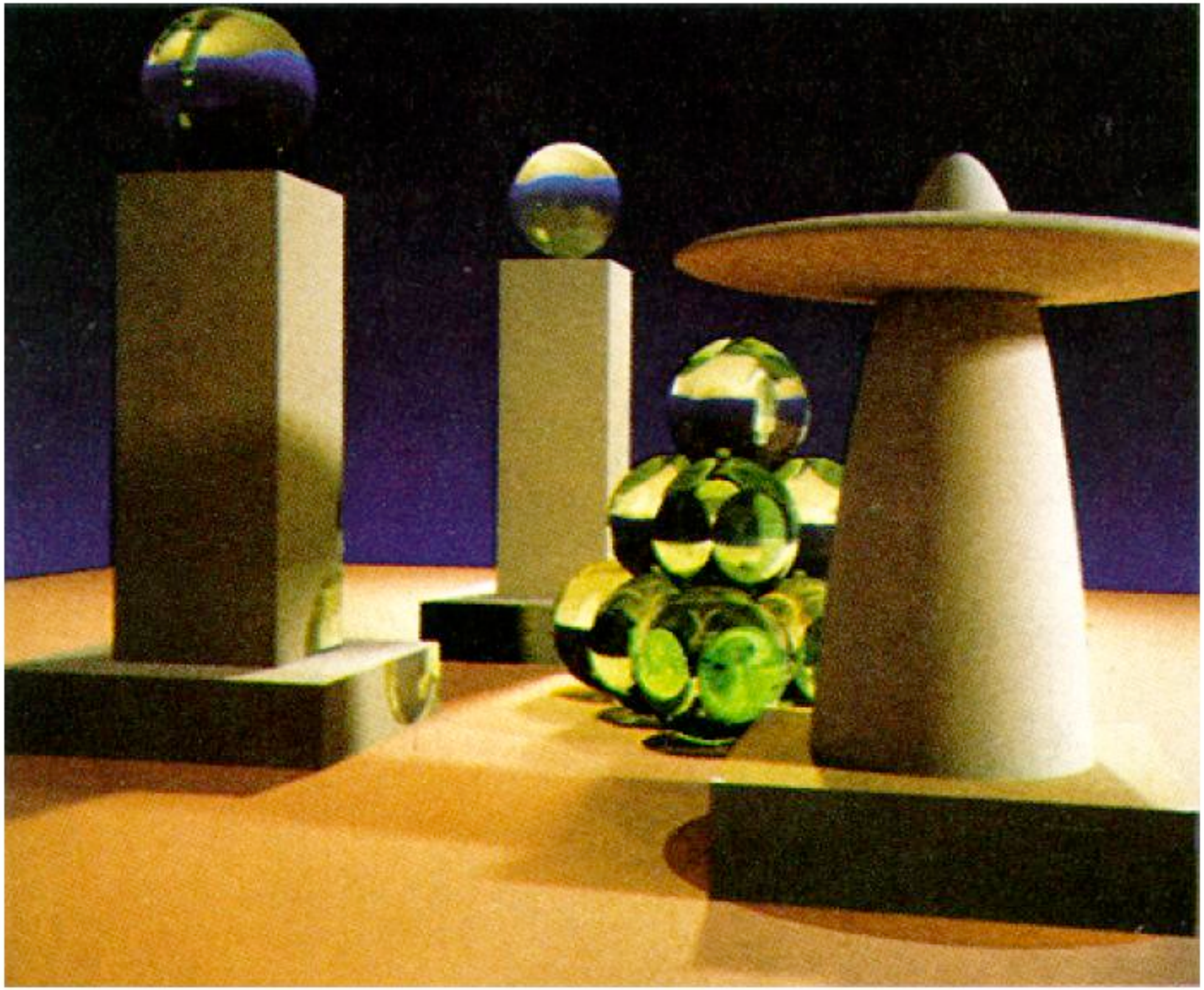


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

"The Rendering Equation" [Kajiya 1986]

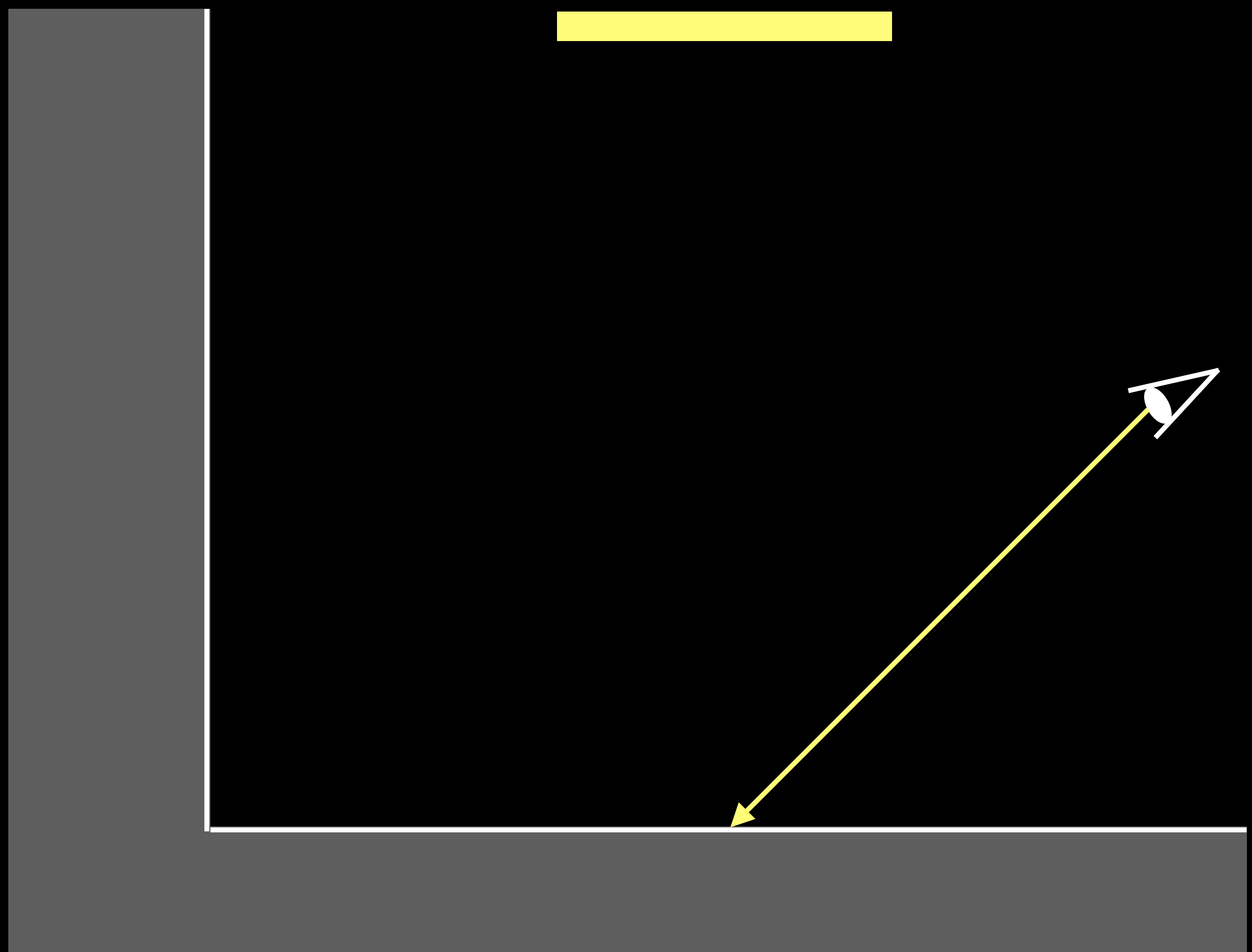
Path Tracing

- Recursive expansion by random sampling

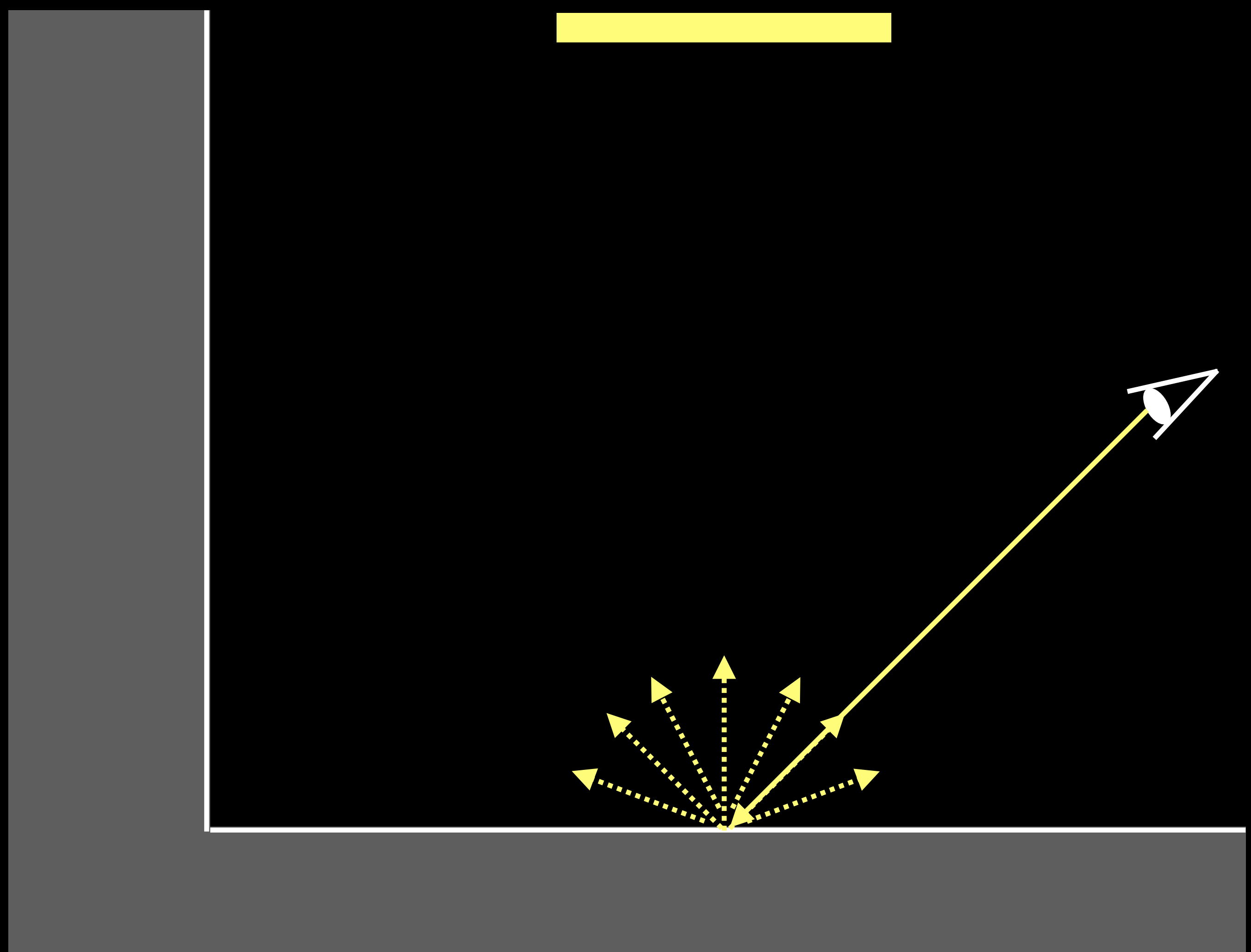
$$L(x \to e) = L_i(x \to e) + \int_\Omega f(\omega, x \to e) L(\omega) \cos\theta d\omega$$

$$L(x \to e) \approx L_i(x \to e) + \frac{f(\omega_0, x \to e) L(\omega_0) \cos\theta_0}{p(\omega_0)}$$

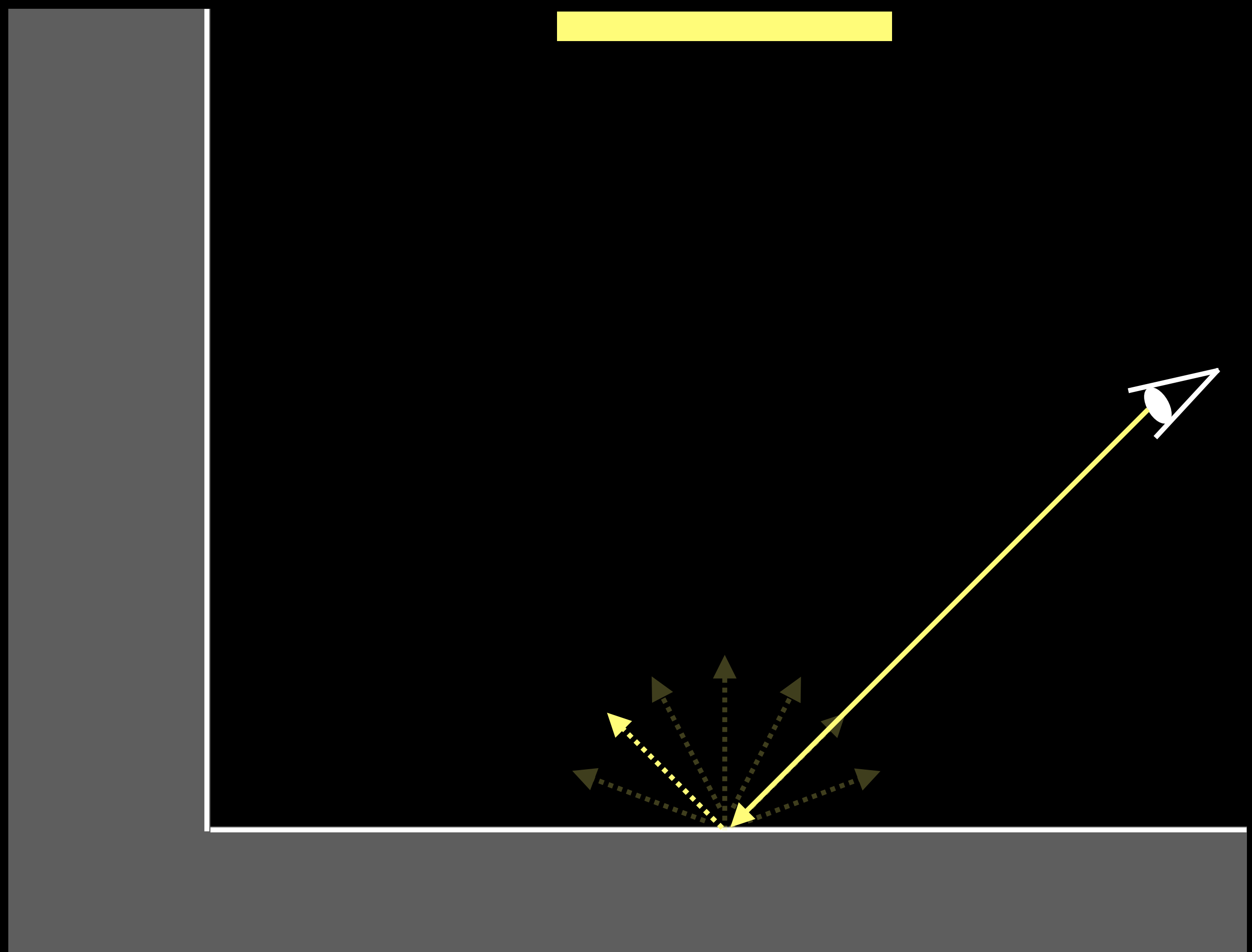
Path Tracing



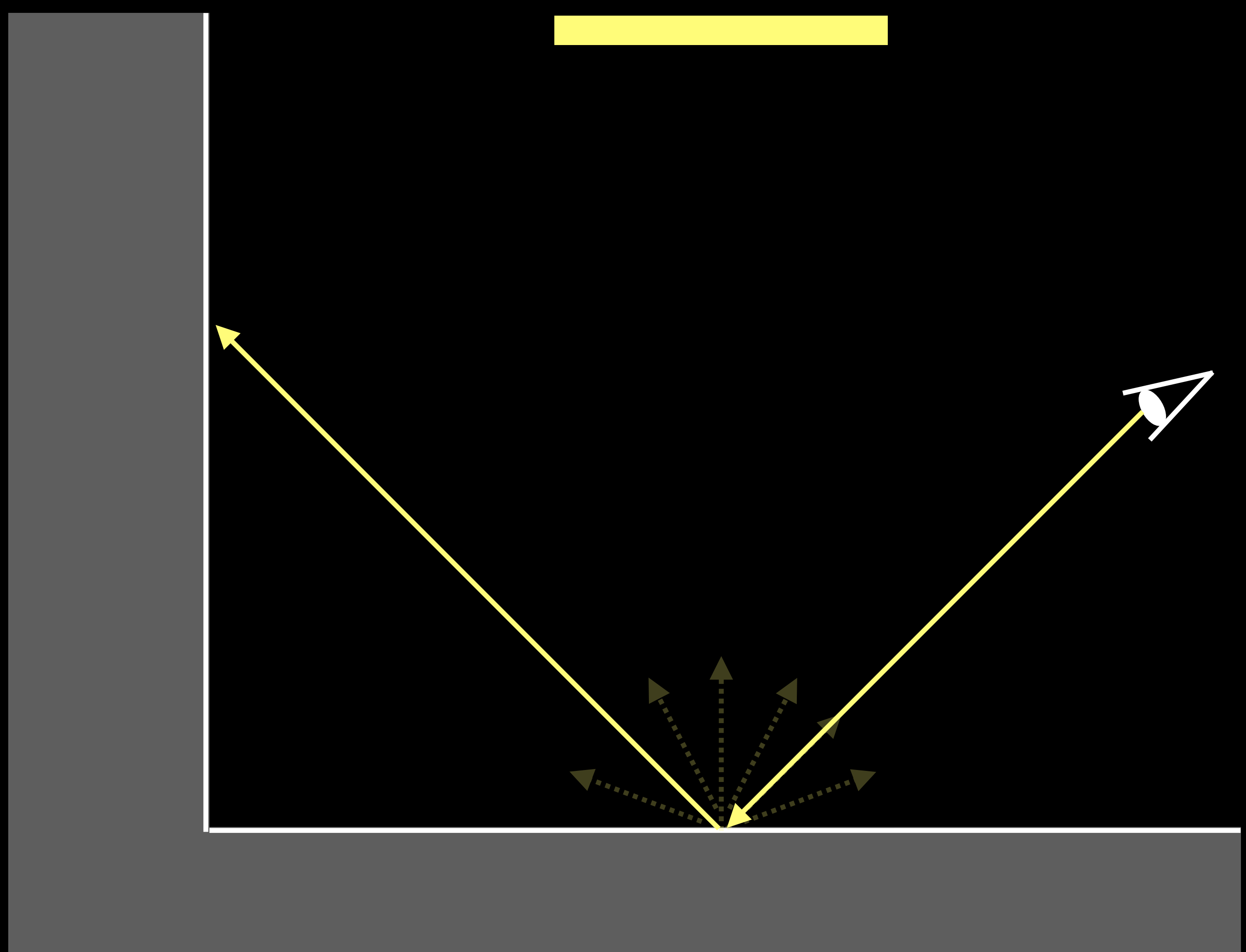
Path Tracing



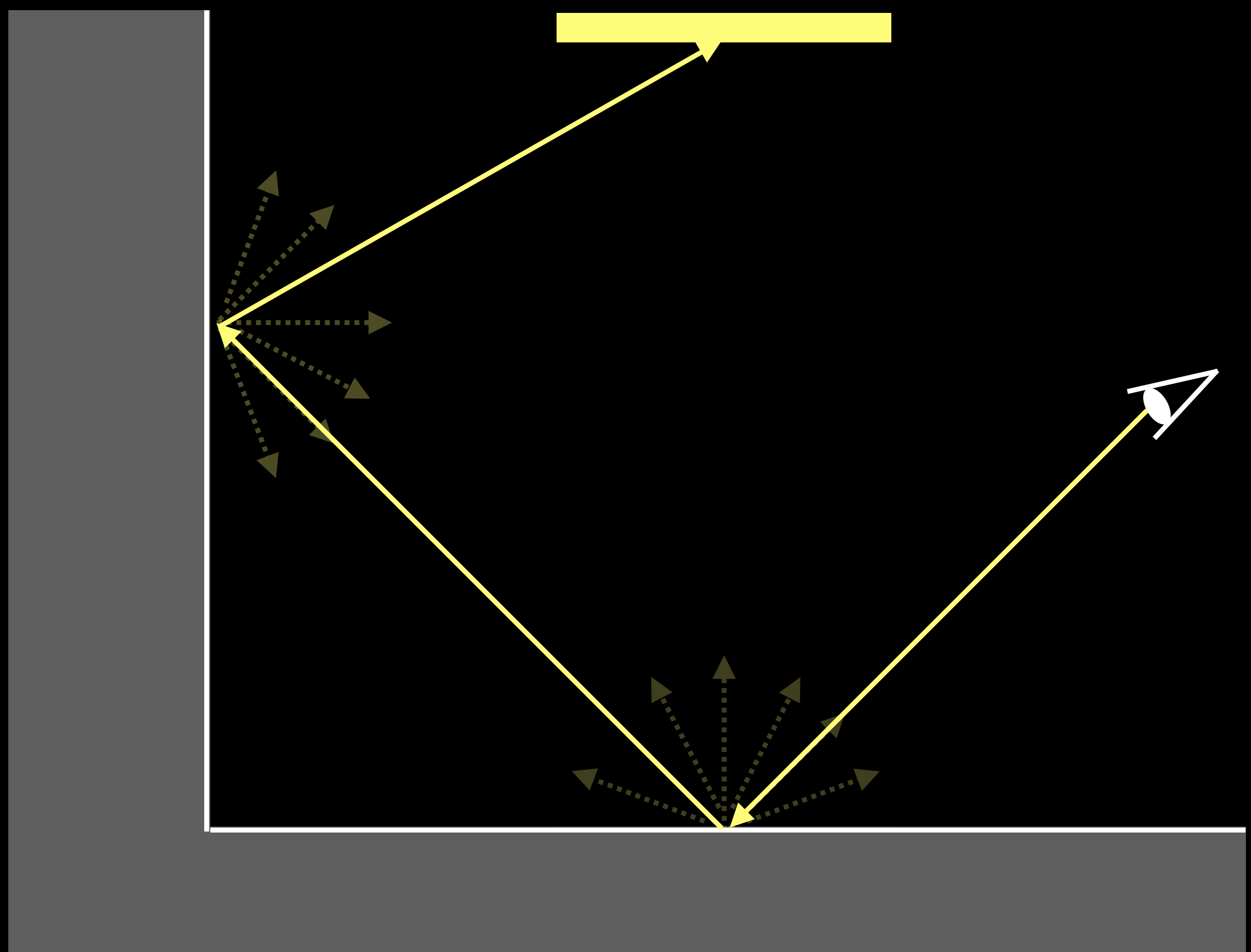
Path Tracing



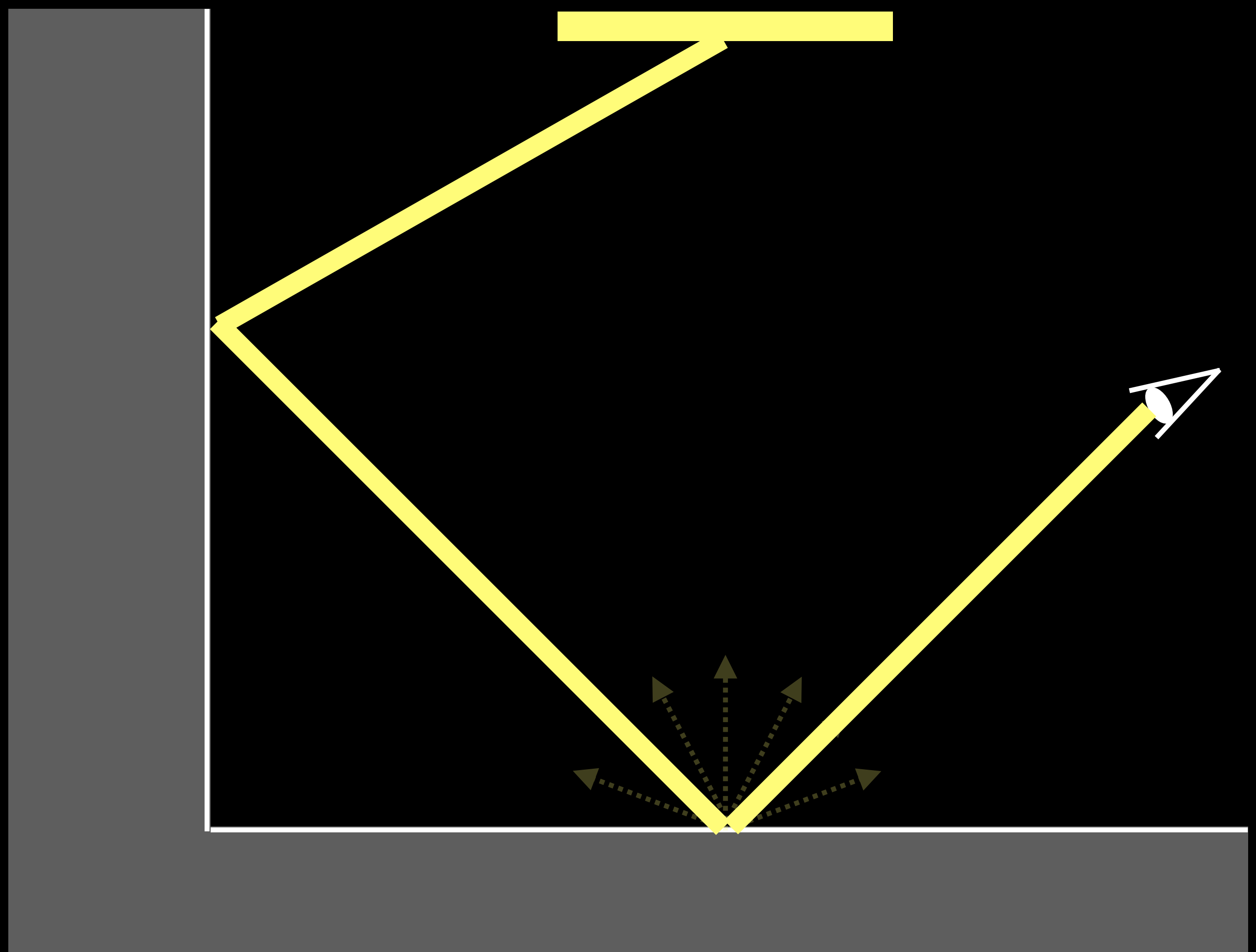
Path Tracing



Path Tracing



Path Tracing



Path Tracing

- Recursive call even for Lambertian
  - Use random directions around the normal
  - Add the emission term for light sources

```
color shade (hit) {
  return (Kd / PI) * get_irradiance(hit)
}
```

Path Tracing

- Recursive call even for Lambertian
  - Use random directions around the normal
  - Add the emission term for light sources

```
color shade (hit) {
  w = random_dir(hit. normal)
  c = max(dot(w, hit.normal), 0)
  d = ray(hit.position, w)
   return Le + (Kd / PI) * shade(trace(d)) * c / p(w)
}
```

Generating Random Directions

- Use spherical coordinates (then get xyz)

$$p(\omega_i) = \frac{1}{2\pi}$$

$$\theta = \arccos(u_1)$$
$$\phi = 2\pi u_2$$

$$u_1, u_2 \in [0, 1]$$

Generating Random Directions

- θ and ϕ are around the normal, not y axis
- Use an orthonormal basis (similar to eye rays)

$$\vec{n}_y = \vec{n}$$

$$\vec{n}_x = \frac{\vec{c} \times \vec{n}}{|\vec{c} \times \vec{n}|} \qquad \vec{\omega}_i = x\vec{n}_x + y\vec{n}_y + z\vec{n}_z$$

$$\vec{n}_z = \vec{n}_x \times \vec{n}$$

$\vec{c}$: a vector that is not parallel to normal

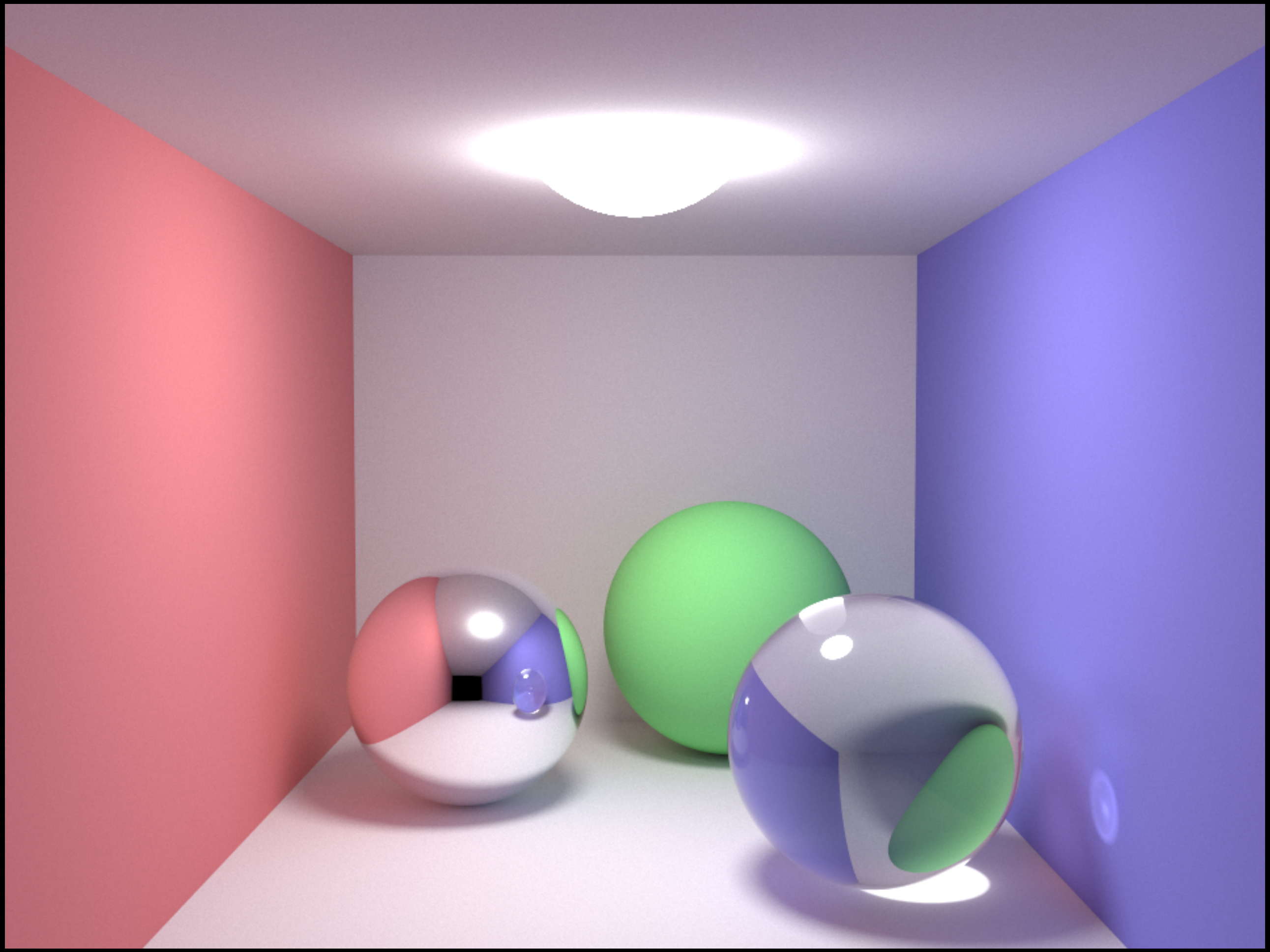
Path Tracing

- Take the average of random samples

```
for all pixels {
    ray = generate_camera_ray( pixel )
    first_hit = traverse( ray, accel_data_struct )
    pixel = 0
    for i = 1 to N {
      pixel = pixel + shade( first_hit )
    }
    pixel = pixel / N
}
```

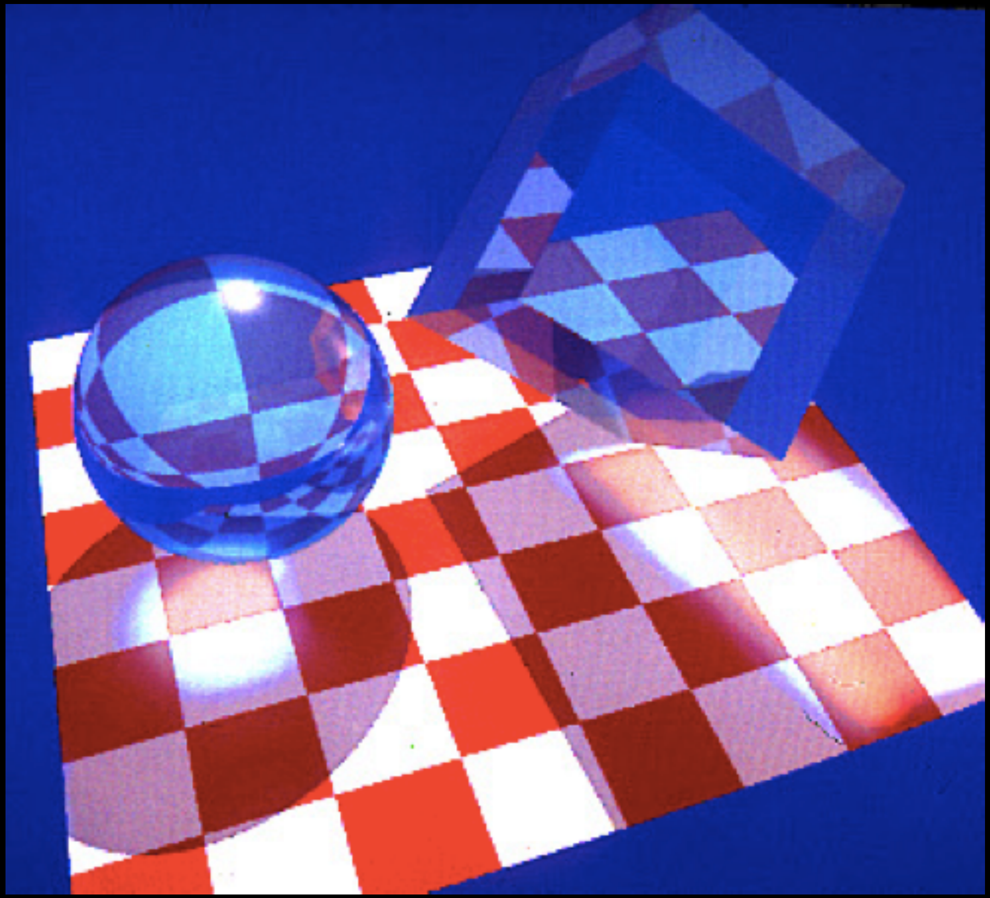
Example

- edupt (http://kagamin.net/hole/edupt/)



Light Tracing

- Trace paths from light sources



http://courses.cs.washington.edu/courses/cse457/14sp/projects/trace/extra/Backward.pdf

Bidirectional Path Tracing

- Trace paths from both light sources and eye

(a) Bidirectional path tracing with 25 samples per pixel

(b) Standard path tracing with 56 samples per pixel (the same computation time as (a))

https://graphics.stanford.edu/courses/cs348b-03/papers/veach-chapter10.pdf

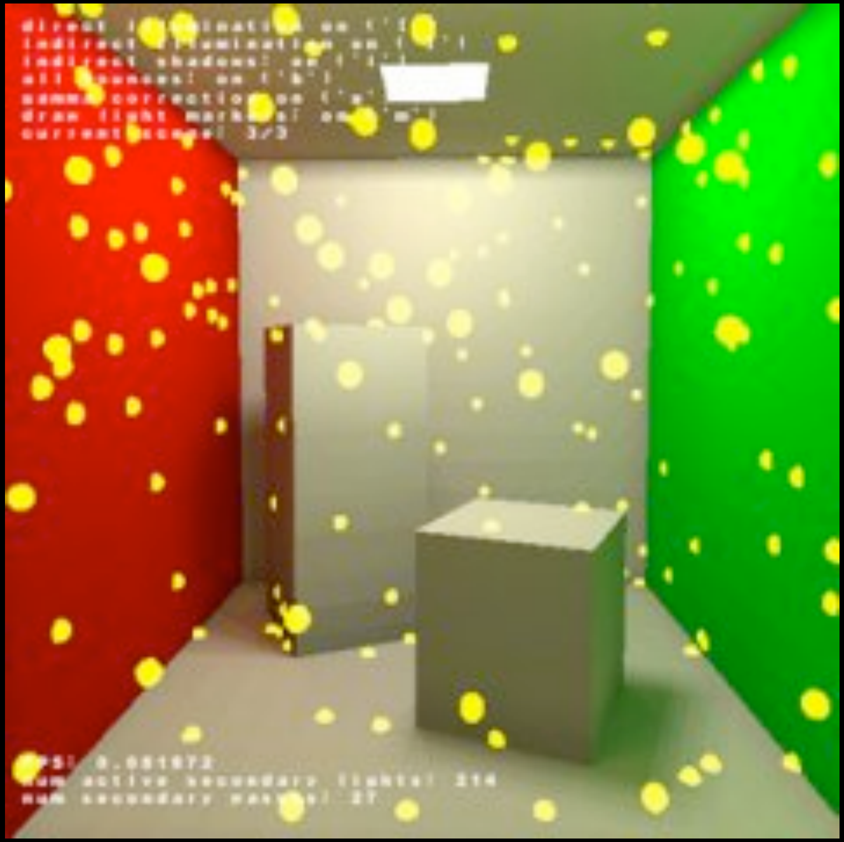
Markov Chain Monte Carlo

- Generate a new sample by mutating the old

https://graphics.stanford.edu/papers/metro/metro.pdf

Many Lights Methods

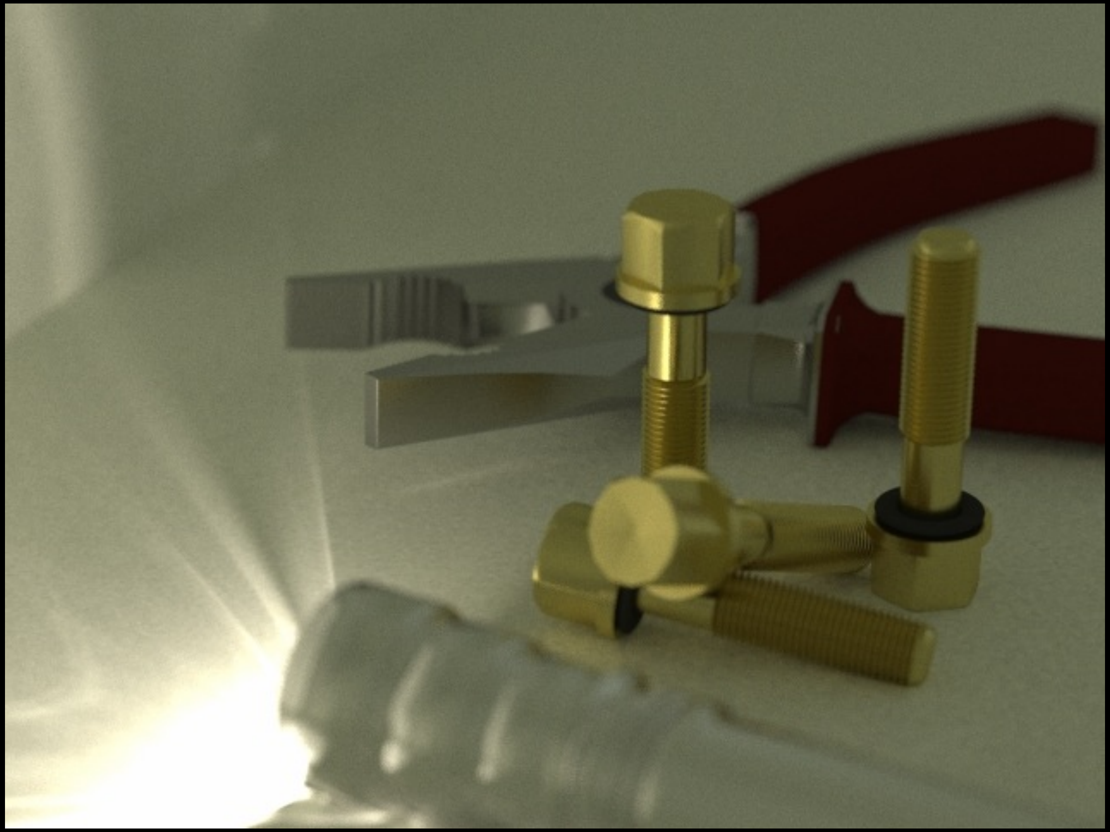
- Lots of point lights for simulating diffuse bounces



http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.40.2213&rep=rep1&type=pdf

Photon Density Estimation

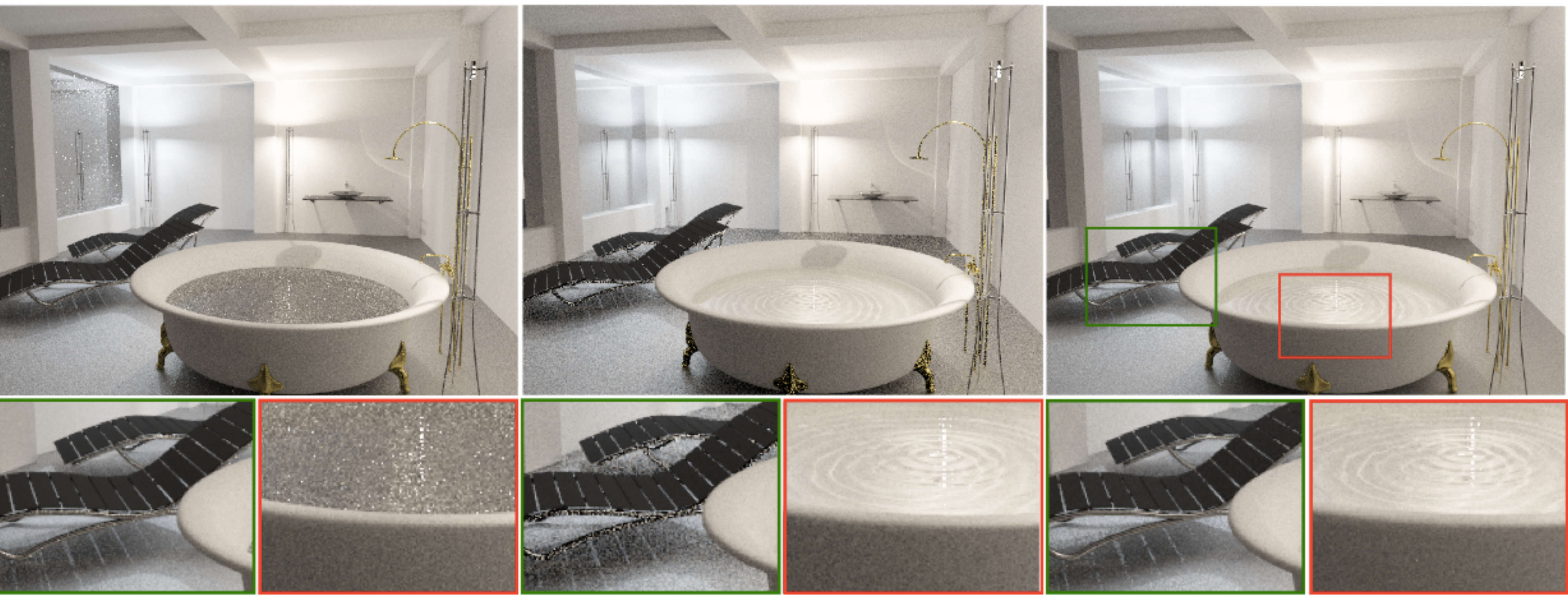
- Estimate densities of path vertices



https://cs.uwaterloo.ca/~thachisu/starpm2013a/

Bidirectional Path Tracing + Photon Density Estimation

- Automatically combine two algorithms



Monte Carlo Path Integration (RMSE: 0.07836) Photon Density Estimation (RMSE: 0.04638) Unified Framework (RMSE: 0.01246)

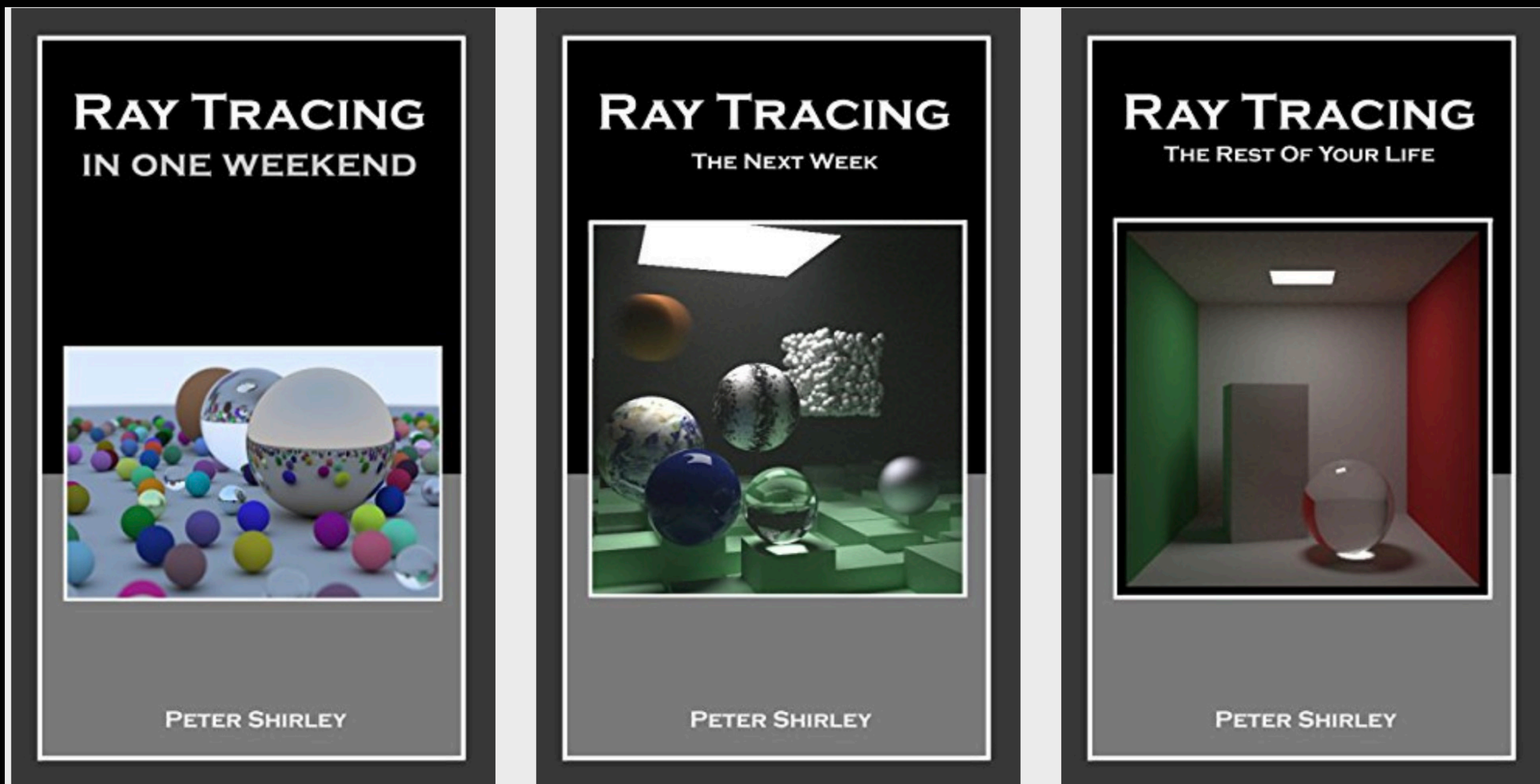
https://cs.uwaterloo.ca/~thachisu/ups.pdf

pbr-book.org

- Freely available online textbook
- "Literate programming"
- https://github.com/mmp/pbrt-v3/

Ray Tracing in One Weekend

by Peter Shirley



https://raytracing.github.io/